Multi User Beam Selection Using Sequential Competition Test

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***Abstract*—We present a novel sequential hypothesis test for multi user beam selection when knowledge about the SNR operating point of each user is not available. The proposed multi user sequential competition test reliably selects the strongest beam for each user while adaptively changing the average test length (the number of observations) according to the SNR operating point of each user. The performance of the proposed algorithm in terms of overall SINR is almost identical to the performance of the ideal (genie) beam selector when the packing ratio or the number of users over number of beams is small.**

***Index Terms*—Multi User Beam Selection, Sequential Test, Generalized Likelihood Ratio Test**

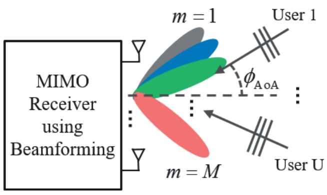

Fig. 1. Illustration of the beam selection problem for a single path channel: Which beam should be chosen to capture the highest amount of signal power?

I. Introduction

To serve multiple users via the same time and frequency resource, the users must be separated spatially. Directional data transmission in practical multiple antenna systems can be simply achieved via electronically controllable beamforming networks like a Butler Matrix [1]. Such a network provides a set of orthogonal beams (denoted as 'codebook') steered into different mainlobe directions.

The crucial question at the receiver side, showed schematically in Fig 1, is how to choose the best beam for each user from a given codebook such that the overall Signal to Interference plus Noise Ratio (SINR) is maximized. The conventional approach [2] is to estimate the captured power of each user under each beam using a training sequence of *fixed* length and then to choose the beam for each user with the highest SINR estimate. To use this method efficiently, i.e. not with a much larger/smaller number of observations than necessary to achieve the desired performance level, knowledge of the Probability Density Functions (PDFs) of all estimates would be required. However, this knowledge is not or only roughly available [3] to the detector due to varying operation conditions under which this problem needs to be solved.

We have presented in [4][5][6] a novel M*-ary* sequential test for beam selection when knowledge about the SNR operating point is *not* available. The proposed sequential test adaptively changes the test length (the number of observations) according to the SNR operating point to achieve the desired performance. Moreover, to achieve the same performance in terms of captured signal power, it requires on average less observations (particularly in the lower SNR regime) in comparison to a perfectly tuned fixed length test assuming genie knowledge. In this work we extend the idea of beam selection based on sequential competition for a multi-user scenario. This algorithm learns on the fly the current statistics of the signals corresponding to each user in terms of their amplitudes and noise variances based on the available observations from each beam. In this way the decision on the best beam(s) for each user can be made both as early as possible and adaptively according to the SNR operating point.

II. Multi-user Beam Selection Problem

Consider the problem of allocating beams from an orthogonal codebook to different users such that they are spatially separated as much as possible. Assuming a flat rank one channel[1] (i.e. a single path) for each user in the uplink, the complex valued received samples are observed separately under each candidate beam as

$$r_m[n] = \sum_{u=1}^{U} a_{u,m} s_u[n] + w_m[n] \ , \qquad (1)$$

where $m \in \{1, \ldots, M\}$, $n \in \{1, \ldots, N\}$ and $u \in \{1, \ldots, U\}$ indicate the beam, sample and user indices. The parameter $a_{u,m}$ denotes the combined effective channel and beamforming gain corresponding to user u under beam m, which is treated as a deterministic unknown complex amplitude. $w_m[n]$ is complex zero mean white Gaussian noise (WGN) sample with unknown variance σ^2 under beam m. A pseudo-random (PN) sequence with $s_u[n] \in \{\pm 1\}$, variance one and $P\{s_u[n] = +1\} = P\{s[n] = -1\} = 1/2$ is assumed for training of each user so that $\mathrm{E}[s_u[n] s_u[n-k]] \simeq \delta[k]$ holds for its autocorrelation sequence, while the crosscorrelation between different sequences corresponding to different users is $\mathrm{E}[s_u[n] s_{u'}[n-k]] \simeq 0$ when $u \neq u'$.

[1]Although the channel model for mmWave systems contain features like multipath with multiple delays and angle of arrivals, we believe this simplistic channel model suffices for analyzing the optimal detection strategy.

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In order to allocate the best beam to each user during the training interval, the receiver should evaluate the SINR value $\rho_{u,m}$ for each user u under each beam m as

$$\hat{\rho}_{u,m} = \frac{|\hat{a}_{u,m}|^2}{\sum_{i=1,i\neq u}^{U} |\hat{a}_{i,m}|^2 + \hat{\sigma}^2} \ , \tag{2}$$

where the hat denotes the estimated value. Using the SINR estimates $\hat{\rho}_{u,m}$, the most promising (u, m) pairs can be decided for via exhaustive search, which involves prohibitively high complexity with $\binom{M}{U}$ searches. On the other hand, the accuracy of the Minimum Variance Unbiased Estimators (MVUEs) [7] of the parameters $a_{u,m}$ and σ^2 based on training sequences of fixed length N, depends on the SNR operating point at which these values are estimated. However, this information is hidden from the receiver. Therefore fixing the test length (i.e. sequence length) can lead to drastically changing performance when the information about the SNR operating point of each user is not available. Additionally, if N is conservatively set to a high value based on the worst still acceptable operating point, a lot of time that is spent for detection of the best beam for each user, will be wasted, if the channel quality is actually better than expected. This is particularly important when the channel coherence time is limited and wasting time for training results in loss of throughput. These problems can be potentially avoided by using an adaptive variable length test.

We explain in the next section the Generalized Likelihood Ratio Test (GLRT) for the classical linear model and will use this result to introduce our *multi user sequential competition test* in Section IV.

III. Generalized Likelihood Ratio Test for A Linear Model with Unknown Noise Variance

Consider that the data follows the linear model $\boldsymbol{r} = \boldsymbol{S}\boldsymbol{A} + \boldsymbol{w}$, where $\boldsymbol{S}$ is a known $N \times U$ $(N > U)$ matrix of rank U, $\boldsymbol{A}$ is a $U \times 1$ vector of unknown parameters, and $\boldsymbol{w}$ is a $N \times 1$ noise vector with PDF $\mathcal{N} \sim (\boldsymbol{0}, \sigma^2\boldsymbol{I})$ while σ^2 is unknown.

The Generalized Likelihood Ratio (GLR) for hypothesis testing problem

$$\begin{array}{ll} \mathcal{H}_0 & : \boldsymbol{B}\boldsymbol{A} = \boldsymbol{b}, \ \sigma^2 > 0 \\ \mathcal{H}_1 & : \boldsymbol{B}\boldsymbol{A} \neq \boldsymbol{b}, \ \sigma^2 > 0 \end{array} \ , \tag{3}$$

where $\boldsymbol{B}$ is a $K \times U$ matrix $(K \leqslant U)$ of rank K, $\boldsymbol{b}$ is a $K \times 1$ vector, can be written as

$$L(\boldsymbol{r}) = \frac{p\left(\boldsymbol{r}; \hat{\boldsymbol{A}}_{\mathcal{H}_1}, \hat{\sigma}^2_{\mathcal{H}_1}\right)}{p\left(\boldsymbol{r}; \hat{\boldsymbol{A}}_{\mathcal{H}_0}, \hat{\sigma}^2_{\mathcal{H}_0}\right)} = \left(\frac{\hat{\sigma}^2_{\mathcal{H}_1}}{\hat{\sigma}^2_{\mathcal{H}_0}}\right)^{N/2} \ , \tag{4}$$

where

$$\begin{aligned} \hat{\boldsymbol{A}}_{\mathcal{H}_1} &= (\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S})^{-1}\boldsymbol{S}^{\mathrm{H}}\boldsymbol{r} \\ \hat{\sigma}^2_{\mathcal{H}_1} &= (\boldsymbol{r} - \boldsymbol{S}\hat{\boldsymbol{A}}_{\mathcal{H}_1})^{\mathrm{H}}(\boldsymbol{r} - \boldsymbol{S}\hat{\boldsymbol{A}}_{\mathcal{H}_1}) \\ \hat{\boldsymbol{A}}_{\mathcal{H}_0} &= \hat{\boldsymbol{A}}_{\mathcal{H}_1} - (\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S})^{-1}\boldsymbol{B}^{\mathrm{H}}(\boldsymbol{B}(\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S})\boldsymbol{B}^{\mathrm{H}})^{-1}(\boldsymbol{B}\hat{\boldsymbol{A}}_{\mathcal{H}_1} - \boldsymbol{b}) \\ \hat{\sigma}^2_{\mathcal{H}_0} &= (\boldsymbol{r} - \boldsymbol{S}\hat{\boldsymbol{A}}_{\mathcal{H}_0})^{\mathrm{H}}(\boldsymbol{r} - \boldsymbol{S}\hat{\boldsymbol{A}}_{\mathcal{H}_0}) \ , \end{aligned}$$

are Maximum Likelihood Estimates (MLEs) of $\boldsymbol{A}$ and σ^2 under $\mathcal{H}_1$ and $\mathcal{H}_0$ respectively, while $(.)^{\mathrm{H}}$ denotes conjugate transpose operation.

It can be shown [7] that the modified GLR statistic $\gamma(\boldsymbol{r})$ follows the central and non-central F distribution under $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively, as

$$\gamma(\boldsymbol{r}) = \frac{N-U}{K}\left(L(\boldsymbol{r})^{2/N} - 1\right) \sim \begin{cases} F_{K,N-U}, & \mathcal{H}_0 \\ F'_{K,N-U}(\lambda), & \mathcal{H}_1 \end{cases} \ , \tag{5}$$

where the non-centrality parameter is

$$\lambda = \frac{(\boldsymbol{B}\boldsymbol{A} - \boldsymbol{b})^{\mathrm{H}}(\boldsymbol{B}(\boldsymbol{S}^{\mathrm{H}}\boldsymbol{S})^{-1}\boldsymbol{B}^{\mathrm{H}})^{-1}(\boldsymbol{B}\boldsymbol{A} - \boldsymbol{b})}{\sigma^2} \ .$$

This results in the following threshold criterion in terms of GLRT

$$\gamma(\boldsymbol{r}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma_{\mathrm{th}} \ . \tag{6}$$

The exact detection performance in terms of probability of false alarm (P_{FA}) and probability of detection (P_{D}) is given by

$$P_{\mathrm{FA}} = Q_{F_{K,N-U}}(\gamma_{\mathrm{th}}) \tag{7}$$

$$P_{\mathrm{D}} = Q_{F'_{K,N-U}(\lambda)}(\gamma_{\mathrm{th}}) \ , \tag{8}$$

where $Q_{F_{K,N-U}}$ ($Q_{F'_{q,N-U}(\lambda)}$) returns the right tail probability of a central (non-central) F distribution with K degrees of freedom in the nominator and $N - U$ degrees of freedom in the denominator. Since now the PDF of $\gamma(\boldsymbol{r})$ under $\mathcal{H}_0$ is completely known, one can ensure that P_{FA} will not surpass a predefined value by finding a proper threshold γ_{th} in Eq. 7.

IV. Multi User Beam Selection Based on Sequential Competition

The received sequence under beam m mentioned in Eq. (1) after observing n samples, follows the linear model which can be written as

$$\boldsymbol{r}_m = \boldsymbol{S}\boldsymbol{A}_m + \boldsymbol{w}_m \ , \tag{9}$$

where $\boldsymbol{S}$ is the known $n \times U (n > U)$ training matrix including all training sequences corresponding to different users up to sample n, $\boldsymbol{A}_m = [a_{1,m}, \ldots a_{u,m}]^{\mathrm{T}}$ is the $U \times 1$ vector of unknown complex amplitudes corresponding to different users under beam m and $\boldsymbol{w}_m$ is a $n \times 1$ vector of complex WGN noise samples with unknown variance under beam m.

Similar to our single user *sequential competition test* in [6], instead of comparing the estimates of the captured power of all users under different beams with each other, we rather decompose the $M - ary$ test for each user into M parallel binary test w.r.t a virtual no signal hypothesis. Hence, the following binary test for user u under beam m can be formulated as

$$\begin{array}{ll} \mathcal{H}_{u,m,0} & : \boldsymbol{B}_u\boldsymbol{A}_m = 0, \ \sigma^2 > 0 \\ \mathcal{H}_{u,m,1} & : \boldsymbol{B}_u\boldsymbol{A}_m \neq 0, \ \sigma^2 > 0 \end{array} \ , \tag{10}$$

where $\boldsymbol{B}_u$ is defined as a $1 \times U$ vector with all entries equal to zero except the u^{th} entry corresponding to the user u which is set to one. The binary test based on decision metric $\gamma_u(\boldsymbol{r}_m)$

implies that for each user u under each beam m, we check the presence of a signal corresponding to that user against its absence. Obviously, $\mathcal{H}_{u,m,0}$ is the wrong hypothesis for user u under beam m, assuming that some signal is observable but with different strength. On the other hand, $\mathcal{H}_{u,m,0}$ acts as a virtual common reference in the set of M parallel binary tests for user u. This results in total to $M \times U$ parallel binary tests.

We let all users compete to distinguish their signals from no signal hypothesis under each beam while n can grow until a decision criterion is fulfilled. Let us denote the probability of selecting the presence of the signal corresponding to user u under beam m after n observations as $P_{\mathcal{H}_{u,m,1}}(n)$. Comparing the binary tests of users u under beams m and m', it follows from Eq. (8) that for $|a_{u,m}| > |a_{u,m'}|$ and a common decision threshold γ_{th}, we have $P_{\mathcal{H}_{u,m,1}}(n) > P_{\mathcal{H}_{u,m',1}}(n)$. This is simply a consequence of the fact that the accumulated deflection coefficient (equivalent to non-centrality parameter) $\lambda_{u,m}$ will grow more quickly than $\lambda_{u',m}$ as n grows. Therefore, the beam that sees the stronger signal will on average cross the threshold earlier. This fact leads to the following sequential competition test applied to stochastic paths $\gamma_u(\boldsymbol{r}_m)$ for $m = 1, \dots, M$ and $u = 1, \dots, U$ as

$$\gamma_u(\boldsymbol{r}_m(n)) \underset{\text{undecided}}{\overset{\mathcal{H}_{u,m,1}}{\gtrless}} \gamma_{\text{th}} \ , \tag{11}$$

where at each step n all stochastic paths $\gamma_u(\boldsymbol{r}_m(n))$ for $m \in \{1, \dots, M\}$ and $u \in \{1, \dots, U\}$ are compared to the fixed common threshold γ_{th}. As soon as one of the paths surpasses the threshold, the corresponding pair (u, m) is selected and the the training stops for that user while other users continue. The same procedure repeats by taking the next observation into account until all users surpass the threshold or the maximum test length is reached.

The interpretation is that for each user, we let the beams compete to distinguish themselves from pure zero mean WGN with unknown variance and the one which does it faster is the winning beam for the corresponding user in the competition. As a result the overall test length n for each user is now a random variable with $\bar{n}$ as the average number of observations per user.

The multi user sequential competition test selects the strongest beam for each user as early as possible adaptively with respect to the SNR operating point of each user. For low user density, i.e. small values of the ratio U/M which we denote as the packing ratio, the probability that strongest beam for different users overlap, is small and therefore the selected (user,beam) pairs result in near optimal performance in terms of overall SINR. However, this result may not be optimal (specially in higher SNR regime) when the packing ratio U/M is large since multiple users might have the same strongest beam. In order to combat the heavy interference in this case, the affected users may be served via time sharing while using the same beam. The downside to time sharing might be the fact that some RF chains may be wasted since they will not have any contribution to the sum-rate performance. This can be addressed by interference-aware beam allocation for colliding users [8][9].

V. Numerical Evaluation

We numerically studied the performance of the *multi user sequential competition test* in the reference channel model described in Eq. (1) with a uniform linear array with 32 antenna elements using the codebook of a Butler matrix with 32 orthogonal beams with normalized magnitudes. The users have the same normalized power at the receiver before beamforming equal to 1. The AoA was distributed uniformly in $[-90^\circ, 90^\circ]$ independently for each user over the simulation runs while SNR was defined as $1/\sigma^2$. The average SINR values were estimated at each SNR point based on 10^4 simulation runs for SNR values in the interval $[-6, 6]$ dB. For comparison we consider the ideal beam selector based on genie knowledge on $a^2_{u,m}$ and σ^2 values. As shown in Fig. 2, the multi user sequential competition test with γ_{th} based on $P_{\text{FA}} = 10^{-3}$, shows a very close performance in terms of average SINR per user compared to the ideal beam selector based on genie knowledge. This demonstrates the robustness of the sequential test under SNR variations.

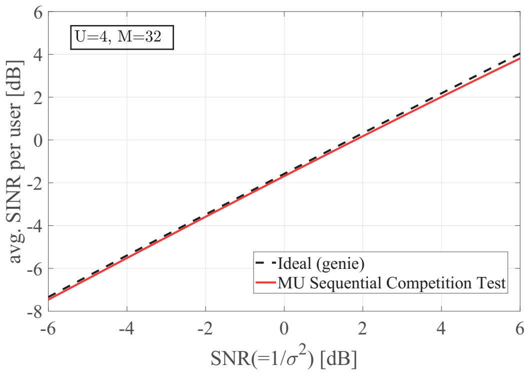

Fig. 2. Comparison on average SINR per user after ideal beam selection based on genie knowledge versus MU sequential competition test.

This feature is achieved by adaptively changing the average test length per user $\bar{n}$ with respect to the SNR operating point as illustrated in Fig. 3.

VI. Concluding Remarks

We proposed a novel sequential hypothesis test based on GLR statistics to solve the composite multi user beam selection problem. The proposed *multi user sequential competition test* selects the strongest beam for each user as early as possible adaptively with respect to the SNR operating point of each user. The achieved performance in terms of average overall SINR per user is close to the performance of the ideal beam selector based on genie knowledge on angle of arrivals and noise variance for small packing ratios. Similar behaviors can be observed in a more complicated multipath mmWave channel which requires additional correlation steps at different delays.

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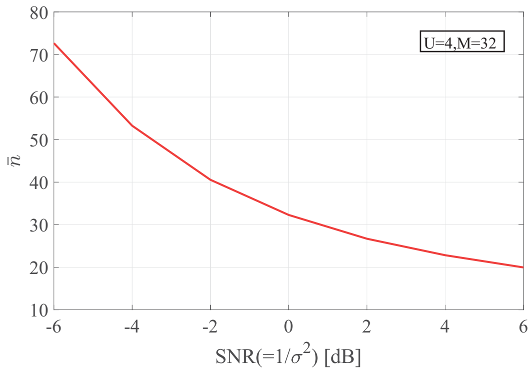

Fig. 3. Average number of observations $\bar{n}$ per user over different SNR values resulted from MU sequential competiton test .

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