Optimal SDNR Digital Predistortion via Direct Inversion

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Abstract—Digital Predistortion (DPD) is a technique used to compensate for nonlinear distortion in RF Power Amplifiers (PAs). The Indirect Learning Architecture (ILA) is a commonly used method for identifying DPD coefficients. However, ILA produces biased estimates, leading to sub-optimal results in the presence of measurement noise. Additionally, the DPD coefficients obtained with ILA depend on the input signal used for identification, which requires a new identification process for different input signals. In this context, the question arises: is it possible to estimate DPD coefficients that fulfill some optimality criterion? To address this question, an approach is proposed that assumes knowledge of input signal statistics and a static quasi-memoryless polynomial model. The static nature of the model implies that the coefficients do not change with time, while the quasi-memoryless nature indicates that the polynomial coefficients are complex-valued, modeling both AMAM and AMPM. An analytical solution for the DPD coefficients that maximizes the Signal-to-Distortion-and-Noise Ratio (SDNR) is obtained. Simulation results show that our approach outperforms the ILA. Furthermore, since our approach relies on knowledge of the PA models, it is possible to use it to obtain the optimal DPD coefficients for different input signals without the need for a new identification process.

Index Terms—Power Amplifier (PA), Digital Predistortion (DPD), Rayleigh Quotient, Formal Power Series (fps)

I. INTRODUCTION

Digital Pre-Distortion DPD is a well-known technique used in Radiofrequency PAs to compensate for nonlinear distortion. Power amplifiers present the highest energy efficiency point near saturation, the same point where nonlinearity due to saturation is strongest. If nonlinearity is not compensated, the performance of communication systems can be significantly degraded. Nonlinear distortion in power amplifiers results in spectral regrowth, which can cause interference with other communication systems and limit the available bandwidth. If the PAs operate below the saturation region (i.e. in the linear region, technique known as “back-off”), the energy efficiency of the PA is reduced. DPD is used to mitigate this effect by predistorting the input signal to cancel out the nonlinear distortion introduced by the power amplifier.

Several methods have been proposed to design DPD systems, including the Indirect Learning Algorithm ILA [1], which is a widely used method for identifying DPD coefficients. However, the ILA has limitations, such as being a biased estimator [2], which leads to sub-optimal results in the presence of high measurement noise. Additionally, the DPD coefficients obtained with ILA depend on the input signal used for identification, requiring a new identification process for input signals with different statistics.

The research problem addressed in this paper is the lack of an optimal design of the DPD system that can efficiently mitigate nonlinear distortion while minimizing the impact of measurement noise and input signal changes. The objective of this paper is to propose a methodology to obtain DPD parameters that optimize the Signal-to-Distortion-and-Noise Ratio (SDNR). To achieve this objective, the proposed approach assumes knowledge of input signal statistics and a static quasi-memoryless polynomial model to estimate the DPD coefficients that fulfill the optimality criterion. By obtaining DPD parameters that optimize the SDNR, the proposed methodology can improve the performance of communication systems while minimizing interference with other systems.

The remainder of this paper is organized in three sections. Section II depicts the theoretical basis of the power matrices to optimize the SDNR, followed by Section III which illustrates the results obtained by the proposed methodology and compares it with state-of-the-art solutions. Finally, Section IV discusses the conclusions of this paper and future work.

Throughout this paper, the following notation is used: a scalar is represented using lowercase letters, $h$; a vector using bold-face, $h$; and a matrix with uppercase bold-face, $H$. Conjugate, transpose, and hermitian transpose are represented as $(\cdot)^\ast$, $(\cdot)^T$ and $(\cdot)^H$, respectively. To represent the element at row $m$ and column $n$ of a matrix, the notation $[\cdot]_{m,n}$ is used; and zero based indexing is assumed, i.e. $m,n \in \mathbb{Z}_{+}^T$.

II. PROPOSED METHOD

The analysis relies on representing polynomial composition using Power Matrices, resulting in an elegant formulation that allows to use methods well known for linear systems into the nonlinear domain requiring functional composition.

A. Power Matrix Fundamentals for Polynomials

In this section the algebra of polynomial composition is presented. The goal is to show how polynomial composition can be presented as a matrix multiplication, via the isomorphism between polynomials and matrices. Only the very fundamentals required for this paper will be introduced; for more details refer to [3, Chapter 1].

Consider the fps over the field of complex numbers:

$$f = a_0 + a_1 x + a_2 x^2 + \ldots \, ; \, a_i \in \mathbb{C}$$
where the fps form an integral domain over C. Moreover, the addition between two fps is defined as:
\[ f + g := (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + \ldots \]
and the multiplication via the Cauchy Product:
\[ f \cdot g := (a_0b_0) + (a_0b_1 + a_1b_0)x + (a_0b_2 + a_1b_1 + a_2b_0)x^2 + \ldots \]
The units of this integral domain (i.e., elements that have a multiplicative inverse) are the series with \( a_0 \neq 0 \). Units have properties useful for polynomial multiplication. Elements with \( a_0 = 0 \) are called nonunits. On the other hand, the subset of fps with \( a_0 = 0 \) and \( a_1 \neq 0 \), called almost units, form a group under composition, namely:
- They are closed under composition
- There exists an identity element
- Every almost unit has an inverse under composition:
\[ f \circ f^{-1} = X \]

Furthermore, an infinite matrix \( \mathbb{A} \) can be associated to the fps \( f \):
\[ f \rightarrow \mathbb{A} = \begin{bmatrix} a_1 & a_2 & a_3 & \ldots \\ a_2 \cdot & a_2^2 & a_2^3 & \ldots \\ a_3^2 & a_3^3 & \ldots \\ \vdots & \vdots & \ddots \end{bmatrix} \]
where \( a_n^{(k)} \) represents the factor besides \( x^n \) in \( f^k \):
\[ f^k = \sum_n a_n^{(k)} x^n \]
In other words, every row \( k \) of the Power Matrix \( \mathbb{A} \) contains the coefficients of raising the fps \( f \) to the power of \( k \). This matrix representation is an isomorphism [3], and has the following properties:
- Let \( f \) and \( g \) be nonunits, and let \( f \rightarrow \mathbb{A}, g \rightarrow \mathbb{B} \). Then:
\[ f \circ g \rightarrow \mathbb{A} \cdot \mathbb{B} \]  
- Let \( f \rightarrow \mathbb{A}, g \rightarrow \mathbb{B}, h \rightarrow \mathbb{D} \). The associativity law for composition holds:
\[ (f \circ g) \circ h = f \circ (g \circ h) \rightarrow (\mathbb{A} \cdot \mathbb{B}) \cdot \mathbb{D} = \mathbb{A} \cdot (\mathbb{B} \cdot \mathbb{D}) \]
- The inverse element respect to the composition law can be computed via matrix inversion. Let \( f \rightarrow \mathbb{A} \), then:
\[ f^{-1} \rightarrow \mathbb{A}^{-1} \]
This is called the reversion of the fps \( f \), where the notation \( f^{-1} \) is used to distinguish it from the multiplicative inverse of \( f \). This is highlighted as follows:
\[ f^{-1} = \frac{1}{a_1x + a_2x^2 + \ldots} \]
- The \( n \)th section of the infinite matrix \( \mathbb{A} \) is defined as the finite submatrix \( \mathbb{A}_n \) that contains the first \( n \) rows and columns of \( \mathbb{A} \). Both \( \mathbb{A} \) and \( \mathbb{A}_n \) are upper triangular matrices. The product of the \( n \)th section of two upper triangular matrices is the \( n \)th section of the product of the matrices:
\[ \mathbb{A}_n \cdot \mathbb{B}_n = (\mathbb{A} \cdot \mathbb{B})_n \]

B. PA and DPD models
Quasi-memoryless polynomials are a broadly used model for PA characterization and predistortion [4]. They are defined as:
\[ y = f(x) = \sum_{l=1}^{L} a_l x |x|^l ; \quad a_l \in \mathbb{C} \]
We consider only the odd order terms of the polynomial, since even order terms would require the use of square root function into the base. Moreover, consider the following basis functions:
\[ h(u) : \mathbb{C} \rightarrow \mathbb{C} \]
\[ h(u) = [u \quad u|u|^2 \ u|u|^4 \ldots u|u|^{2L-2}]^T = [u \quad u^2 u^* \ u^3 u^* \ldots u^L u^{(L-1)*}]^T \]
Then, the PA and DPD are modeled by quasi-memoryless polynomials as:
\[
\begin{align*}
\text{PA} : \quad y &= f(x) = \sum_{l=0}^{L-1} a_{2l+1} x |x|^{2l} = \mathbf{a}^T \cdot \mathbf{h}(x) \\
\text{DPD} : \quad x &= g(u) = \sum_{l=0}^{L-1} b_{2l+1} u |u|^{2l} = \mathbf{b}^T \cdot \mathbf{h}(u)
\end{align*}
\]
where \( L \) is the number of coefficients, \( \mathbf{a} \) and \( \mathbf{b} \) are the coefficient vectors, and the PA output is the composition of the two polynomials
\[ y = f(g(u)) = \mathbf{a}^T \cdot \mathbf{h}(g(u)) = \mathbf{a}^T \cdot \mathbf{h}(\mathbf{b}^T \cdot \mathbf{h}(u)) \]
However, expanding the polynomials is tedious (results in a polynomial with degree \( (L(L-1))^2 \)). Instead, the power matrices can be used to compute the PA’s output easily. Notice that:
\[
\begin{align*}
g(u) &= b_1 u + b_3 u^3 u^* + b_5 u^5 u^* + \ldots \\
(g(u))^2(g(u))^* &= \begin{bmatrix} b_1 & b_3 & \ldots \\ b_3 & b_5 & \ldots \\ \vdots & \vdots & \ddots \end{bmatrix} \cdot \begin{bmatrix} b_1 \ b_3^* \ b_5^* \ldots \\ b_3 \ b_5^* \ b_7^* \ldots \\ \vdots & \vdots & \ddots \end{bmatrix} \\
\mathbf{B} &= \begin{bmatrix} b_1 & b_3 & \ldots \\ b_3 & b_5 & \ldots \\ \vdots & \vdots & \ddots \end{bmatrix} \\
\mathbf{E} &= \begin{bmatrix} b_1 \ b_3^* \ b_5^* \ldots \\ b_3 \ b_5^* \ b_7^* \ldots \\ \vdots & \vdots & \ddots \end{bmatrix} \\
x &= g(u) \rightarrow \mathbf{h}(x) = \mathbf{B} \cdot \mathbf{h}(u)
\end{align*}
\]
Similarly, for \( f(x) \):
\[
\begin{align*}
f(x) &= a_1 x + a_3 x^2 x^* + a_5 x^3 x^* + \ldots \\
(f(x))^2(f(x))^* &= \begin{bmatrix} a_1 & a_3 & \ldots \\ a_3 & a_5 & \ldots \\ \vdots & \vdots & \ddots \end{bmatrix} \cdot \begin{bmatrix} a_1^* \ a_3^* \ a_5^* \ldots \\ a_3^* \ a_5^* \ a_7^* \ldots \\ \vdots & \vdots & \ddots \end{bmatrix} \\
\end{align*}
\]

Fig. 1. Signal model: a complex normal input signal $u$ is predistorted, and later amplified. Thermal noise with variance $\sigma_n^2$ is added at the output.

$$u \sim \mathcal{CN}(0, \sigma_u^2) \xrightarrow{DPD} y \sim \mathcal{CN}(0, \sigma_y^2) \xrightarrow{PA} n \sim \mathcal{CN}(0, \sigma_n^2)$$

Then, using (1), the polynomial composition is converted to a matrix multiplication:

$$y = f(g(u)) \rightarrow h(y) = A \mathcal{B} h(u)$$

The vector $h(y)$ contains $y$ and all its powers $y_i u^{i-2}$. However, the PA output is obtained from the first element of the vector. Thus, let $T_1 = [1 \ 0 \ 0 \ldots]^T$ be the first column of the identity matrix. Then,

$$y = T_1^T h(y) = T_1^T A \mathcal{B} h(u) = a^T \mathcal{B} h(u)$$

C. Signal model

The input signal models an OFDM signal, and follows a complex normal distribution: $u \sim \mathcal{CN}(0, \sigma_u^2)$, where the input signal power is $P_u = \sigma_u^2$. The signal is passed through the DPD, and then through the PA. The PA output is corrupted by thermal noise, modeled also as a random variable with complex normal distribution: $n \sim \mathcal{CN}(0, \sigma_n^2)$. The model is presented in Fig. 1.

For computing signal quality metrics over complex valued signals, a straightforward approach is to use the Bussgang decomposition, as will be described in the next section.

1) Bussgang Decomposition: The system PA(DPD($u$)) can be decomposed into a linear part $y_{lin}$ and a nonlinear distortion noise $\nu$ by using the Bussgang Decomposition [5].

$$z = y_{lin} + \nu + n = G_B u + \nu + n$$

From this decomposition, the SDNR can be written as:

$$SDNR = \frac{E[(G_B u)^2]}{E[(\nu + n)(\nu + n)'^*]} = \frac{|G_B|^2 \sigma_u^2}{E[\nu \nu^*] + \sigma_n^2}$$

The Bussgang Gain $G_B$ is the expected value of the slope of the PA output $y$ with respect to the input $u$ [5]. When working with complex numbers, Wirtinger Calculus is used, which treats $u$ and $u^*$ as independent variables [6].

$$G_B = \mathbb{E}\left[ \frac{\partial}{\partial u} y \right] = \mathbb{E}\left[ \frac{\partial}{\partial u} a^T \mathcal{B} h(u) \right] = a^T \mathcal{B} m_u$$

where the vector $m_u$ contains the moments of the derivative of the base function vector $h'(u)$:

$$m_u = \mathbb{E}\left[ [1 \ 2u u^* \ 3u^2 u^{2*} \ 4u^3 u^{3*} \ldots]^T \right]$$

2) Statistics of the Input Signal: In the case of a complex random variable $u$, its magnitude $|u|$ is Rayleigh distributed. Hence, its moments $|u|^k$ are known:

$$u \sim \mathcal{CN}(0, \sigma_u^2) \rightarrow |u| \sim \mathcal{R}(s = \sigma_u / \sqrt{2})$$

$$E[|u|^k] = \sigma_u^k \Gamma(k/2 + 1)$$

where $\Gamma$ is the Gamma function. Hence, the vector $m_u$ can be computed in closed form as:

$$m_u = \left[ \begin{array}{c} (l + 1) \sigma_u^{2l} \\
\frac{1}{2} \sigma_u^2 \\
6 \sigma_u^4 \\
\ldots \end{array} \right]$$

The covariance matrix of the base function vector $h(u)$ can also be computed in closed form:

$$C_u^{NL} = \mathbb{E}[h(u)h(u)^*]$$
$$C_u^{NL}_{m,n} = \mathbb{E}[u^{2(m+n+1)}] = \sigma_u^{2(m+n+1)} \Gamma(m + n + 2)$$

This is a Hankel matrix, which can be written as:

$$C_u^{NL} = \left[ \begin{array}{cccc}
\sigma_u^2 & \sigma_u^4 & \sigma_u^6 \\
\sigma_u^4 & \sigma_u^6 & \sigma_u^8 \\
\sigma_u^6 & \sigma_u^8 & \sigma_u^{10} \\
\sigma_u^8 & \sigma_u^{10} & \sigma_u^{12} \\
\sigma_u^{10} & \sigma_u^{12} & \sigma_u^{14} \\
\ldots & \ldots & \ldots & \ldots
\end{array} \right]$$

3) Metrics, SDNR: With the statistics of the input signal and the two power matrices defining PA and DPD coefficients, the linear, total, and distortion power can be computed. The linear output power can be obtained as:

$$\mathbb{E}[y_{lin} y_{lin}^*] = G_B G_B^* \cdot \mathbb{E}[u u^*] = a^T \mathcal{B} m_u m_u^H \mathcal{B} H a^* \cdot \sigma_u^2$$

The total output power is:

$$\mathbb{E}[y y^*] = \mathbb{E}[a^T \mathcal{B} h(u)] (a^T \mathcal{B} h(u))^*$$
$$= a^T \mathcal{B} \mathbb{E}[h(u) h(u)^*] \mathcal{B} H a^*$$
$$= a^T \mathcal{B} C_u^{NL} \mathcal{B} H a^*$$

The distortion power is:

$$C_D := \mathbb{E}[\nu \nu^*] = \mathbb{E}[y y^*] - \mathbb{E}[y_{lin} y_{lin}^*]$$
$$= a^T \mathcal{B} (C_u^{NL} - \sigma_u^2 m_u m_u^H) \mathcal{B} H a^*$$

With these definitions, the SDNR can be written as:

$$SDNR = \frac{\mathbb{E}[y_{lin} y_{lin}^*]}{C_D + \sigma_n^2}$$
$$= \frac{a^T \mathcal{B} m_u m_u^H \mathcal{B} H a^* \cdot \sigma_u^2}{a^T \mathcal{B} (C_u^{NL} - \sigma_u^2 m_u m_u^H) \mathcal{B} H a^* + \sigma_n^2}$$

D. SDNR Optimization

The goal is to find the coefficients $b$ that maximize the SDNR as defined in (2). Hence, the following optimization problem can be formulated:

$$\max_{b = \mathcal{B}(\mathcal{H})} \frac{a^T \mathcal{B} m_u m_u^H \mathcal{B} H a^* \cdot \sigma_u^2}{a^T \mathcal{B} (C_u^{NL} - \sigma_u^2 m_u m_u^H) \mathcal{B} H a^* + \sigma_n^2}$$
1) Solution: Rayleigh quotient: The following equation is known as “Rayleigh quotient” [7]:

\[ \mathcal{R}(x; A) = \frac{x^H A x}{x^H x} \]

The “Rayleigh quotient” optimization problem consists of maximizing (or minimizing) \( \mathcal{R}(x; A) \). The optimum can be obtained by solving the following eigenvalue problem [7]:

\[ Ax = \lambda x \]

The maximum (minimum) eigenvalue of \( A \) is the maximum (minimum) Rayleigh quotient. The corresponding eigenvector is the solution of the optimization problem.

\[ \lambda_{\text{max}}(A) = \max_x \mathcal{R}(x; A) \]

The approach for solving Eq. (3) is to express the SDNR as a Rayleigh quotient and solve the corresponding eigenvalue problem. To convert it, the Cholesky decomposition is used, similar to [7].

Let \( x = B^H a^* \), then:

\[ \text{SDNR} = \frac{\chi^H M_u' \chi \cdot \sigma_n^2}{\chi^H (C_{NL}^u - \sigma_u^2 M_u' + \sigma_n^2 \frac{1}{\chi} \chi) \chi + \sigma_n^2} \]

Define \( M_u' := M_u' \chi \).

Using: \( \frac{\chi^H \chi}{\chi^H x} = 1 \), the term \( \sigma_n^2 \) can be integrated into the denominator:

\[ \text{SDNR} = \frac{\chi^H M_u' \chi \cdot \sigma_u^2}{\chi^H (C_{NL}^u - \sigma_u^2 M_u' + \sigma_n^2 \frac{1}{\chi} \chi) \chi} \]

Let \( \chi^H \chi = K \), some \( K \). The matrix in the denominator is symmetric and full rank. Hence, it can be decomposed using the Cholesky decomposition:

\[ C_{NL}^u - \sigma_u^2 M_u' + \sigma_n^2 \frac{1}{\chi} \chi = LL^H \]

Rewriting the denominator:

\[ \chi^H (C_{NL}^u - \sigma_u^2 M_u' + \sigma_n^2 \frac{1}{\chi} \chi \chi) \chi = (\chi^H L)(L^H \chi) =: \chi^H \chi \]

where \( \chi \) is introduced as auxiliary variable:

\[ \chi = L^H x \rightarrow L^{-H} \chi = x \]

Then, the SDNR can be written as:

\[ \text{SDNR} = \frac{\chi^H L^{-1} M_u' \chi \cdot \sigma_u^2}{\chi^H \chi} \]

Hence, the vector \( \chi_{\text{max}} \) that maximizes the SDNR is the eigenvector \( \nu_{\text{eig}} \) of \( L^{-1} M_u' L^{-H} \) corresponding to the maximum eigenvalue \( \lambda_{\text{max}} \).

\[ \max \chi \text{SDNR} = \lambda_{\text{max}}(L^{-1} M_u' L^{-H}) \]

\[ \chi_{\text{max}} = \arg \max \chi \text{SDNR} = \nu_{\text{eig, max}}(L^{-1} M_u' L^{-H}) \]

a) Including the Objective Gain: Note that the Rayleigh quotient is scale invariant. Hence, the solution \( \chi_{\text{max}} \) can be scaled by a constant \( c \) without affecting the optimum:

\[ \mathcal{R}(c \cdot \chi; A) = \mathcal{R}(\chi; A) \]

By definition, the factor in the SDNR numerator correspond to the Bussgang Gain:

\[ \chi^H L^{-1} M_u' L^{-H} \chi = |G_B|^2 \]

By setting the following constraint, the objective linear gain can be included in the optimization problem, and the scale of the solution can be fixed:

\[ (c \cdot \chi^H) L^{-1} M_u' L^{-H} (c \cdot \chi) = |G_{\text{obj}}|^2 \]

Solving for \( |c| \):

\[ |c| = \frac{1}{\sqrt{\chi^H L^{-1} M_u' L^{-H} \chi}} |G_{\text{obj}}| \]

Denote \( \chi_{\text{opt}} \) as the optimum solution constrained to an objective gain. It can be obtained as:

\[ \chi_{\text{opt}} = |c| \chi_{\text{max}} = \chi_{\text{max}} \cdot \frac{|G_{\text{obj}}|}{\sqrt{\chi^H L^{-1} M_u' L^{-H} \chi_{\text{max}}}} \]

b) Obtaining the Optimum DPD Coefficients: The optimum \( \chi_{\text{opt}} \) can be obtained from \( \chi_{\text{opt}} \) and \( L \):

\[ \chi_{\text{opt}} = L^{-H} \chi_{\text{opt}} \]

From \( \chi_{\text{opt}} \), the optimum set of DPD coefficients \( b_{\text{opt}} \) can be obtained by solving the following system of non-linear equations:

\[ \chi_{\text{opt}} = \mathbb{B}^H a^* \]

Note that \( \mathbb{B} \) is a triangular matrix, hence, the solution can be obtained iteratively, by forward substitution. For example, for \( L = 3 \), the 3rd section of matrix \( \mathbb{B} \) is:

\[ \mathbb{B}_3 = \begin{bmatrix} b_1 & b_3 & b_2 \\ 0 & b_1^2 b_1^* & 2 b_1 b_1 b_1^* \\ 0 & b_1^3 b_1^{*2} \end{bmatrix} \]

The following system of equations must be solved:

\[ \begin{bmatrix} x_{\text{opt,1}} \\ x_{\text{opt,2}} \\ x_{\text{opt,3}} \end{bmatrix} = \begin{bmatrix} b_1 & 0 & 0 \\ b_3 & b_1^2 b_1^* & 0 \\ b_5 & b_1^3 b_1^{*2} & b_1 b_1 b_1^* \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \]

For example, solving row by row results in: \( b_1 = x_{\text{opt,1}}/a_1 \), then \( b_3 = \frac{x_{\text{opt,3}} - b_1^3 b_1^{*2} a_2}{a_3} \), etc.
Fig. 2. Input power sweep results for (left) the proposed Optimum SDNR, and (right) the ILA approach. Predistorters with $L$ coefficients from 2 to 7 were used, and compared to the back-off case (black).

Fig. 3. PA model utilized for the simulations, including amplitude-to-amplitude and amplitude-to-phase distortion.

III. EXPERIMENTAL DATA AND RESULTS

In this section a comparison between the proposed method and DPD based on the conventional ILA is presented. The simulations are Monte Carlo based, where complex random samples are generated, and the input signal power $P_{in}$ is swept by varying the parameter $\sigma_u$. A quasi-memoryless PA model is used, presented in Fig. 3. This model was obtained via circuit simulation of a Class AB amplifier [8].

For the experiment, the number of coefficients $L$ is varied, and a DPD is identified for each case using the Optimum SDNR approach. A second predistorter with the same number of coefficients is identified using the ILA approach. In this case, for each power level, a new predistorter is identified using $N = 200$ complex samples. A new set of samples is generated and simulated for measuring SDNR. The SDNR is then measured for each case, and the results are presented in Fig. 2. Both cases are compared to the case of using the PA without predistortion, i.e. the power sweep is similar to using back-off linearization, and the sweep is used to find the point of maximum SDNR.

In Fig. 2, for the Optimum SDNR it can be seen that: for increasing $L$, the point of maximum SDNR across input power increases accordingly. The maximum SDNR for $L = 7$ lies around $P_{in} = -12dB$; in comparison the optimum for ILA lies around $P_{in} = -15dB$. This is of significance, since the energy efficiency also increases with input power. Inspecting the ILA results, it can be observed that the improvement in SDNR for increasing $L$ is not steady: for $L = 7$ the highest SDNR point on the plot is lower than for the others $L$ values.

In Fig. 5 the cases for $L = 7$ are plotted on the same axis, for easier comparison. It can be seen that our approach achieves $8dB$ better SDNR, or around $6dB$ improvement when compared to against the best ILA case. Compared to the best back-off case, the proposed approach results in $13dB$ higher SDNR.

The different signal components resulting from the Bussgang decomposition are investigated in Fig. 4, where we present total output power, Bussgang gain, linear output power, non-linear distortion power, and thermal noise power. It can be seen that our proposed method keeps the distortion power just below the thermal noise, up to $P_{in} = -10dB$, where the PA saturation starts, and the distortion power increases. In comparison, in the ILA case the distortion power start increasing at $P_{in} = -18dB$.

As for the limitations of our formulation, it is worth noting that it is only applicable within a specific range of the input signal. The polynomial-based PA model used in our approach does not include a saturation function, which means that operating outside the signal range where the model was identified can result in unreliable results. This is evident in the performance degradation observed around the point $P_{in} = -5dB$ in Fig. 2.

IV. CONCLUSION

The Power Matrices approach presented in this paper provides a novel and elegant formulation for DPD, considered by the authors as a significant contribution. By converting the nonlinear DPD problem into a linear one, this method
provides an optimum SDNR solution using well-known linear optimization methods. Moreover, the proposed method includes external noise sources such as thermal noise, or could even be helpful for over-the-air identification. Additionally, the solution is expressed using complex coefficients, making it capable of handling AMAM and AMPM. By including input signal statistics, this method provides a solution that can be well-suited for MIMO systems, where many equal PAs are used, but statistics of the input signals on each branch can be very different, due to spatial-peak-to-average-power-rate [9].

Future investigations could include the inclusion of memory and DC consumption in the SISO formulation, as well as a MIMO formulation. Overall, the Power Matrices approach has the potential to significantly improve DPD performance and enhance the efficiency of communication systems.

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REFERENCES