OFDMA with Resource and Traffic Constraints: Sum Rate Maximization with no CSI

Thomas Deckert and Gerhard Fettweis
Vodafone Chair Mobile Communications Systems
Technische Universität Dresden, 01062 Dresden, Germany
{deckert,fettweis}@ifn.et.tu-dresden.de

***Abstract*— For a specific high-throughput wireless local area network based on OFDM we study the sum rate of time and frequency division multiple access (TDMA, OFDMA). In allocating resources we consider a minimum size of the smallest resource unit that a user can be allotted. We further assume packet switched traffic with most of it carried by only a few large packets. Relatively little data is carried by a large number of small packets. We show that given the resource and traffic constraints the sum rate may be drastically reduced compared to the unconstrained case in TDMA. We also show that by allowing many users to be active simultaneously OFDMA maintains high throughput. In OFDMA we can do time-sharing between two different bandwidth allocations. We derive the number and rates of users in each of the modes to maximize the sum rate.**

I. Introduction

In this paper we consider the multiple access scenario, i. e., multiple users want to transmit messages to one common receiver. Many researchers have considered the capacity of the multiple-access scenario (for an overview see [1], [2] and references therein). In this work we focus on obtaining the sum rate for TDMA and OFDMA if constraints on the rates both, in an average and maximum sense, apply. These constraints are motivated by the packet size distribution of the traffic typically carried by the system. For the multi-antenna case with complete channel knowledge at the transmitters this has been the rough topic of [3] where the authors follow a smart queuing theoretic approach.

In this paper we consider the single-antenna case where the transmitters know the channel statistics only. Moreover, we deal with the minimum resource granularity inherent in each system, i. e., the minimum amount of data that can be transmitted in one transmit opportunity. We consider a particular wireless local area network (WLAN) system based on orthogonal frequency division multiplexing (OFDM) being developed by the German research project WIGWAM studying a 1 Gbit/s air interface [4].

In the next section we summarize the features of WLAN traffic and derive two rate constraints that will be important to our analysis. We state in general terms the sum rate maximization to be solved for each multi-access scheme. In Section III we describe the physical layer. Then we carry out the sum rate maximization for TDMA in Section IV and for OFDMA in Section V. We summarize in Section VI and argue that OFDMA outperforms TDMA in our application.

II. Traffic Model and Problem Statement

In WLAN data flows in packets and it is known from a multitude of measurements, e. g., [5]–[7], that the traffic is carried by packets of very disparate size. Roughly, the packet size distribution can be regarded as bi- or tri-modal, i. e., there are only two to three typical packet sizes. Also, while the bulk data is carried by large packets they are few in number compared to much smaller packets which are due to control signaling (e. g., acknowledgments) and low-rate user traffic.

We base this work on a slightly simplified packet size distribution with two traffic classes – one to model a stream of large packets of high rate transmissions that we call the *H class*, the other corresponding to low rate traffic in small packets – the *L class*. Now we can characterize the traffic by two typical packet sizes, S_H and S_L, and corresponding typical rates, R_H and R_L. If η_H, η_L give the fraction of packets in the H and L class then $R_H \sim \eta_H S_H$ and $R_L \sim \eta_L S_L$ with the same proportional constant. Of special significance to the following discussion will be the relative L rate ρ, defined as the ratio between the accumulated L rate R_L and the system sum rate $R_S = R_L + R_H$,

$$\rho = \frac{R_L}{R_S} = \frac{\eta_L S_L}{\eta_H S_H + \eta_L S_L}. \tag{1}$$

Inspired by the measurements cited above we consider L packets of size 50 bytes and H packets of size 1500 bytes and ρ to be in the $0.2 \ldots 0.6$ range typically.

We aim to analyze what effect the parameters ρ, S_L and S_H have on the overall throughput of the system. In particular, we are interested in how to best exploit the rates offered by the physical layer while mapping the traffic to it. The result strongly depends on the physical layer itself and on the multiple access scheme. Thus, we consider the specific OFDM-based physical layer described in [4].

Currently, in WLANs packets are time-multiplexed though typically via random access methods. For the system laid out in [4] a user with favorable channel conditions may be able to transmit several hundred bytes in a single OFDM symbol. But as stated before, lots of packets are considerably smaller than that. With stringent latency constraints a user cannot wait for additional packets in order to aggregate them. This is especially a concern in a multiple access scenario. Then, the user is not able to exploit the rate offered by the physical layer.

This motivates the introduction of a *maximum requested rate* for a class, defined as the rate that is necessary to transmit a single packet in exactly one OFDM symbol, i. e.,

$$\hat{r}_L = \frac{S_L}{M}, \quad \hat{r}_H = \frac{S_H}{M}, \tag{2}$$

where M denotes the number of sub-carriers. In a slight misuse of terms by "rate" we mean "spectral efficiency", i. e., data rates averaged over the total observation time T and available bandwidth B measured in bits per complex dimension ("bits/dim"). A complex dimension corresponds to one use of a sub-carrier channel.

Summarizing, our objective is to determine what multiple access scheme, TDMA or OFDMA, has higher throughput in our system scenario. We define as the throughput the maximum sum rate, R_S, given that the total system energy is constrained and rate restrictions apply. Thus, for each scheme we need to solve the optimization problem

$$R_S = \max_{\mathbf{\Theta}} \sum_{u=1}^{U} R_u(\mathbf{\Theta}), \tag{3}$$

where R_u denotes the rate of user u averaged over T and B and the optimization is over the vector parameter $\mathbf{\Theta}$. Varying $\mathbf{\Theta}$ is subject to the following constraints:

(i) *Constant system energy.* Equivalently, the signal-to-noise ratios (SNRs) of the users averaged over T and B sum up to a constant γ_{S}. We call γ_{S} the system SNR.

(ii) *Average relative L rate.* We aim to guarantee an average relative L rate ρ as defined in (1). We map the H and L classes to at least one "logical" user per class.

(iii) *Maximum requested rates.* Corresponding to their typical packet sizes the L and H users can efficiently exploit an offered rate only if it is at most equal to the maximum requested rate, $\hat{r}_L$ and $\hat{r}_H$, respectively. Allocating a larger rate to a user would not increase the effective throughput as it just wastes resources (time, bandwidth or energy).

Sections IV, V recast the problem for TDMA and OFDMA.

III. System Model

We now introduce the setup of our multiple access system and the channel model.

Both multiple access schemes are considered ideal, i. e., there are no guard intervals or guard sub-carriers necessary, and we assume symbol and frequency synchronous reception of all users. U users share an available bandwidth B and a transmission (frame) duration T. In TDMA the entire bandwidth is used exclusively by user $u \in \mathcal{U} = \{1, \ldots, U\}$ for a duration $\lambda_u T$, $0 \leq \lambda_u \leq 1$, $\sum_u \lambda_u = 1$. In OFDMA a frequency band of $\mu_u B$, $0 \leq \mu_u \leq 1$, $\sum_u \mu_u = 1$ is used exclusively by user u during the entire time T. Each user gets a block of adjacent sub-carriers, as "interleaved" sub-carrier assignments or frequency hopping require rather complex synchronization of the users in practice.

As we consider discrete time and frequency, T corresponds to N temporal samples and B to M sub-carriers. The system performs power control such that the total energy received during time T is constrained. This makes the schemes use the same amount of resources (duration, bandwidth, energy).

We model the channel as being constant during the transmission of one code word (block fading). It may be frequency selective but is frequency-flat on each sub-carrier. We assume the transmitters do not know the instantaneous channel state. Hence they fix their rate and power for the entire transmission.

IV. TDMA Reference Case

In this section we adapt (i)-(iii) to obtain the sum rate of TDMA as reference for our later discussion of OFDMA. In our case it suffices to consider multiplexing $U = 2$ users because

- we have two traffic classes that determine the requested rates (by rate ratio $R_L/R_H = \rho/(1-\rho)$ and maximum values $\hat{r}_L$, $\hat{r}_H$) and
- the users know the average channel statistics only and are power controlled such that multi-user diversity cannot be exploited as in [8].

Thus, the active users of the same traffic class are indistinguishable and are collectively considered as a "logical" user representing that class. We denote the user corresponding to the L class by index L and the H class user by index H.

To maximize the sum rate we adapt the powers and durations of the users' transmissions. With user SNRs γ_L and γ_H as well as $\lambda_L = 1 - \lambda_H$ we have the optimization parameter $\mathbf{\Theta} = [\gamma_L, \gamma_H, \lambda_L]$. The energy constraint (i) becomes

$$\lambda_L \gamma_L + (1 - \lambda_L)\gamma_H \leq \gamma_{\mathrm{S}}. \tag{4}$$

The average user rates, needed in (3) and (ii), depend on the SNR and relative duration of the user, i. e., $R_L(\mathbf{\Theta}) = R_L(\gamma_L, \lambda_L)$ and $R_H(\mathbf{\Theta}) = R_H(\gamma_H, 1 - \lambda_L)$, as

$$R(\gamma, \lambda) = \lambda f(\gamma, 1, p). \tag{5}$$

Here, $f(\gamma, 1, p)$ is the maximum rate a single user can achieve with SNR γ using the entire bandwidth during its active time λT. Furthermore, p is the outage probability, i. e., with probability p the user faces a channel that does not support the fixed rate $f(\gamma, 1, p)$ of the user. More formally, we have

$$f(\gamma, \mu, p) = \arg\max_{r} \left\{ \Pr\left(C(\gamma \boldsymbol{\kappa}, \mu) < r \right) < p \right\}, \tag{6}$$

where the instantaneous capacity of a single link conditioned on a particular channel state is [9]

$$C(\gamma \boldsymbol{\kappa}, \mu) = \frac{1}{\mu M} \sum_{m=1}^{\mu M} \log_2 \left(1 + \kappa(m)\gamma \right). \tag{7}$$

In (6), (7) μ is the relative bandwidth of the user such that μM is integer. $\boldsymbol{\kappa}$ is the μM-dimensional channel power vector with, in general, correlated entries $\kappa(m)$ with $\mathrm{E}[\kappa(m)] = 1$. (As all blocks of μM sub-carriers have the same statistics it does not matter in (6) which one we select.)

Finally, we consider condition (iii). An active user gets to transmit at least one OFDM symbol. Without packet aggregation the rate $\hat{r}$ to transmit a single packet in just one OFDM symbol is the maximum rate the user can exploit. We call $\hat{r}$

the *maximum requested rate*. If the user was granted a higher rate it still could not transmit more data (in fact, there may be no more data available). Thus, (iii) can be rewritten as

$$f(\gamma_L, 1, p) \leq \hat{r}_L,$$
$$f(\gamma_H, 1, p) \leq \hat{r}_H. \quad (8)$$

Note that by assuming the medium access layer not to do packet aggregation we consider the worst case. However, this is relevant in applications like network-gaming that have stringent delay constraints such that packet aggregation might be impossible after all. Furthermore, by invoking this assumption we can clearly separate the physical layer-related effects from mechanisms of the higher layers.

Now, given a system SNR γ_{S}, relative L rate ρ and maximum requested rates $\hat{r}_L$ and $\hat{r}_H$, we can obtain R_S based on (3) constrained by (4), (1) and (8). Consider first that (8) is not active. Then the maximum sum rate is that of a single user with SNR γ_{S}, $R_S = f(\gamma_{\mathrm{S}}, 1, p)$, which is achieved by setting equal the user SNRs, $\gamma_L = \gamma_H = \gamma_{\mathrm{S}}$. This holds regardless of ρ and is a consequence of $f(\gamma, 1, p)$ being concave in γ.

For too large γ_{S} constraint (8) becomes active. As $\hat{r}_L < \hat{r}_H$ the L user will be restricted first. Then the power granted to the L user can be reduced such that its instantaneous rate equals $\hat{r}_L$, and thus $\gamma_L < \gamma_{\mathrm{S}}$ is fixed with $f(\gamma_L, 1, p) = \hat{r}_L$. As long as the H user is not restricted itself it may use more power such that $\gamma_H = (\gamma_{\mathrm{S}} - \lambda_L \gamma_L)/(1 - \lambda_L) > \gamma_{\mathrm{S}}$. Now the maximization of R_S depends on λ_L only and reduces to

$$R_S = \max_{\lambda_L} \left\{ \frac{\lambda_L \hat{r}_L}{\rho} \right\} \quad (9)$$

$$\text{s.t.} \quad (1-\rho)\lambda_L \, \hat{r}_L = \rho \, (1-\lambda_L) \, f\left(\frac{\gamma_{\mathrm{S}} - \lambda_L \gamma_L}{1 - \lambda_L}, 1, p \right).$$

Note that R_S according to (9) is based on unequal user SNRs which is suboptimal for maximizing R_S given system SNR γ_{S}. Therefore $R_S < f(\gamma_{\mathrm{S}}, 1, p)$ in this case.

Eventually, at some γ_{S}^* the H user is restricted as well, i. e., with λ_L^* solving (9) we have $f\left((\gamma_{\mathrm{S}}^* - \lambda_L^* \gamma_L)/(1 - \lambda_L^*), 1, p\right) = \hat{r}_H$. At γ_{S}^* the sum rate is maximal, $R_S^* = (\rho/\hat{r}_L + (1-\rho)/\hat{r}_H)^{-1}$. Increasing the SNR further will not increase throughput as no more packets can be mapped to the physical layer resources.

Example: TDMA throughput for Gbit WLAN system. For the system in [4] Fig. 1 shows the throughput of TDMA for varying system SNR γ_{S} at fixed relative L rate $\rho = 0.17$. The maximum sum rate $R_S(\gamma_{\mathrm{S}})$ is normalized with respect to the maximum rate of a single user, $R(\gamma_{\mathrm{S}}, 1)$ (see (5). With $M = 596$ sub-carriers as in [4] and packet sizes of 50 and 1500 bytes we have $\hat{r}_L = 0.67$ bits/dim and $\hat{r}_H = 20.13$ bits/dim, respectively. The channel has an IEEE 802.11n channel D power delay profile and 100 MHz bandwidth. The outage probability is $p = 0.01$.

In Fig. 1 we see that for low SNR, $\gamma_{\mathrm{S}} \lesssim 2$ dB, TDMA achieves the maximum rate as the maximum rate constraint (8) is inactive, i. e., as long as $R_S(\gamma_{\mathrm{S}}) < \hat{r}_L$. For larger SNR values the rate loss due to mismatching small packets to OFDM symbols becomes dramatically apparent. The maximum rate achieved if (8) is active in both users is $R_S^* = 3.45$ bits/dim which is attained for SNRs beyond practical relevance. We will return to the remaining curves in the next section.

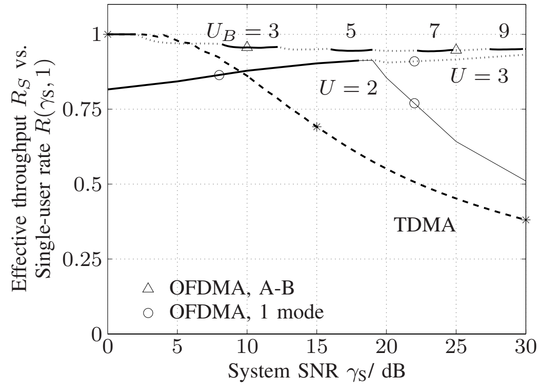

Fig. 1. Normalized throughput for varying γ_{S}, $R_L/R_S = 0.17$, and $\hat{r}_L = 0.67$ bits/dim.

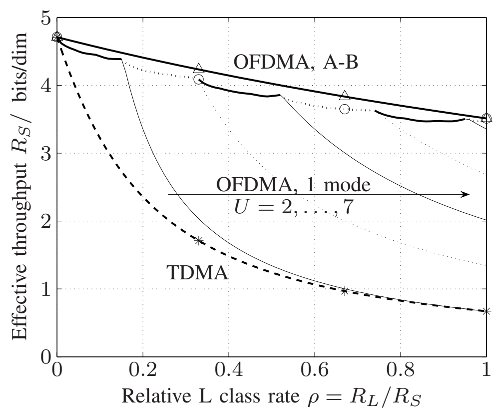

Fig. 2. Throughput for varying relative L rate R_L/R_S, $\gamma_{\mathrm{S}} = 20$ dB and $\hat{r}_L = 0.67$ bits/dim. In OFDMA-AB $U_B = 6$ and $U = 1 + U_B = 7$.

Fig. 2 shows R_S for fixed $\gamma_{\mathrm{S}} = 20$ dB and varying ρ. As is intuitively clear the rate loss becomes more severe the larger the relative L rate, i. e., the relative number of small packets, gets. The huge gap between the single-user reference (at $R_S = 4.7$ bits/dim) and TDMA motivates considering another multiple access scheme.

V. OFDMA

OFDMA makes possible simultaneous transmission of several users. This reduces the number of bits a user can transmit per OFDM symbol compared to TDMA. Therefore the maximum rate constraints (iii) become less stringent and we should see an increased sum rate. Again, we begin by adapting the constraints to OFDMA before analyzing the sum rate.

In OFDMA the maximum rate that a user can exploit depends on the number of sub-carriers it uses. The fewer the number of sub-carriers the higher the rate must be to transmit a single packet in the user's portion of one OFDM symbol. Thus, in contrast to TDMA, it makes sense to consider $U \geq 2$ users. With the user SNRs $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_U]$ and relative bandwidths $\boldsymbol{\mu} = [\mu_1, \ldots, \mu_U = 1 - \sum_{u=1}^{U} \mu_u]$ we form the optimization

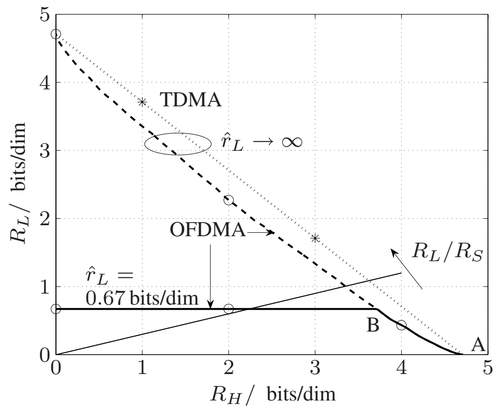

Fig. 3. Two user rate regions for OFDMA, and TDMA unconstrained by (iii). $\gamma_S = 20\,\text{dB}$.

parameter, $\boldsymbol{\Theta} = [\boldsymbol{\gamma}\,\boldsymbol{\mu}]$. The energy constraint (i) becomes

$$\sum_{u=1}^{U} \mu_u \gamma_u \leq \gamma_S. \tag{10}$$

The average rate of a user with SNR γ and bandwidth μB at outage probability p is given by (5) with μ replacing λ and the second argument of f, i. e.,

$$R(\gamma, \mu) = \mu f(\gamma, \mu, p). \tag{11}$$

Depending on the interpretation of (ii) OFDMA can be operated in two ways:

- *Maintain relative L rate at all times.* The system operates in a single mode corresponding to one specific bandwidth and power allocation to a certain number of H and L users. The resource allocation is such that the relative L rate ρ constraint is satisfied with largest possible sum rate. We denote this scheme by OFDMA-1.
- *Relative L rate averaged over some longer time span.* The system may time-multiplex between different modes, e. g., A and B, with resource allocations corresponding to different relative L rates. We denote this scheme by OFDMA-AB.

We consider OFDMA-1 first as OFDMA-AB builds on it.

OFDMA-1. To gain insight into OFDMA-1 operation in the presence of constraint (ii) consider the two user OFDMA rate region given a fixed system SNR. It is maximized by choosing equal user SNRs as in TDMA. The region may be obtained as in [10] or by obtaining R_L, R_H from ρ and $R_S(\rho)$. The relevant dashed line in Fig. 3 shows an example for $\gamma_S = 20\,\text{dB}$ and the parameters used in Section IV. Observe that the rate region is concave. The reason for this is that with multiple users each is allocated a smaller band compared to a single user. Thus they will loose some frequency diversity and that will reduce their individual rates as well as R_S.

Now we determine how many users should be considered. Note from Fig. 3 that R_S is largest if either R_L or R_H are zero. This implies that to maximize R_S as few users as possible should transmit with their largest possible rate. Due to constraint (ii) there must be at least two. Due to (iii) it may make sense to allow more users to transmit. We see in Fig. 3 that for our practical system the L user class reaches its maximum rate sooner (in terms of ρ and γ_S) than the H user class. Thus, we limit our discussion to the case of U users among which one is an H user and $U-1$ users are L users.

Summarizing, we can rewrite the relative L rate constraint (ii) as

$$(1-\rho)\sum_{u=2}^{U} R_u(\boldsymbol{\Theta}) = \rho R_1(\boldsymbol{\Theta}) \tag{12}$$

and the maximum rate constraint (iii) as

$$R_u(\boldsymbol{\Theta}) = \mu_u f(\gamma_u, \mu_u, p) \leq \begin{cases} \hat{r}_H & u = 1 \\ \hat{r}_L & u \in \{2, \ldots, U\} \end{cases}, \tag{13}$$

where user 1 is the H user and the remaining are L users. Note that here R_u is the rate averaged over all M sub-carriers rather than over time (see Section III). Thus, the maximum rate user u needs to transmit a single packet in one OFDM symbol, $\hat{r}$, must be compared to R_u directly and we have (13). The effect of constraint (13) can be seen in Fig. 3 where the rate region is limited by the heavy solid boundary and much smaller than the unconstrained region.

With (10), (12) and (13) we can obtain R_S from (3) for every choice of U, γ_S and ρ. For the system parameters from Section IV Fig. 1 (circles) shows the normalized sum rate versus γ_S. As already pointed out, reduced frequency diversity per user reduces the sum rate with respect to the single-user maximum, especially for low SNR values. In the low SNR region TDMA is not – or only mildly – affected by the maximum rate constraint (iii). Thus, TDMA beats OFDMA-1 there; but at a low absolute sum rate ($R_S \approx 0.5\,\text{bits/dim}$ at $\gamma_S = 0\,\text{dB}$). For higher γ_S (8) limits TDMA while in OFDMA-1 (13) is not yet active and the relation reverses.

With $U = 2$, at some γ_S^* the maximum rate constraint becomes active in OFDMA-1 also. In our case of $\hat{r}_L \ll \hat{r}_H$ the L user is restricted only, i. e., $R_L = \hat{r}_L$. However, due to (12) the rate of the H user is restricted to $R_H = \hat{r}_L(1-\rho)/\rho$ indirectly. Therefore $R_S = \hat{r}_L/\rho$ regardless of the system SNR and bandwidth allocation, i. e., spending $\gamma_S > \gamma_S^*$ only wastes energy. In our numerical example in Fig. 1 we observe a drop in the normalized sum rate of two users above $\gamma_S^* \approx 18\,\text{dB}$.

In contrast to TDMA adding more users can be beneficial. We add more L users as that user class limits performance. Then, as evident from Fig. 1, the throughput increases further for $U = 3$ with increasing $\gamma_S > \gamma_S^*$. However, at sufficiently high SNR the sum rate is limited by (13) again (not shown in the figure). By accounting for the combined maximum L rate in the U-user case we obtain for the sum rate limited by (13)

$$R_S = \frac{(U-1)\hat{r}_L}{\rho}. \tag{14}$$

Fig. 2 (circles) shows the sum rate versus ρ. For fixed U, as the relative L rate increases more bandwidth should be allocated to the L users. However, at some ρ^* the maximum rate constraint becomes active again. Then (14) describes the sum rate for $\rho > \rho^*$ if the number of users stays at U. The figure demonstrates that adding more users helps maintaining a large throughput even with high relative L rate ρ. However, note that the throughput with $U > 2$ users never exceeds that with $U - 1$, confirming that as few users as possible should transmit. Note also that for the SNR chosen TDMA is always outperformed by OFDMA-1.

OFDMA-AB. For this multi-mode scheme we need to determine the modes i, individually characterized by a relative L rate ρ_i and sum rate $R_S(\rho_i)$, between which we time-share the system operation. We wish to maximize the sum rate $R_S^{AB}(\rho)$ given a target ρ. By time-sharing all rate vectors in the convex hull of the underlying OFDMA-1-rate region can be achieved. Time-sharing makes sense if this region is "locally concave" in the direction of ρ such that the appropriate rate vector on the convex hull corresponds to a sum rate $R_S^{AB}(\rho)$ that is larger than $R_S(\rho)$. In Fig. 3 this would be the case for all ρ for which the OFDMA-1-rate vectors lie on the ("locally concave") segment between A and B. It can also be concluded that for any ρ we need to pick no more than two modes. Moreover, these modes correspond to "corner points" of the OFDMA-1-rate region, i. e., rate vectors where two "locally concave" segments of the rate region meet.

Let the modes be A and B. Without loss of generality $\rho_A < \rho_B$. For $\rho_A \leq \rho \leq \rho_B$ the OFDMA-AB sum rate $R_S^{AB}(\rho)$ is

$$R_S^{AB}(\rho) = \lambda R_S(\rho_A) + (1-\lambda) R_S(\rho_B) \tag{15}$$

$$\lambda = \frac{(\rho_B - \rho) R_S(\rho_B)}{(\rho - \rho_A) R_S(\rho_A) + (\rho_B - \rho) R_S(\rho_B)}.$$

where λ is the relative time the system spends in mode A. In Fig. 3 mode A corresponds to single H user transmission, i. e., $\rho_A = 0$. In mode B one L user transmits at its maximum rate $\hat{r}_L$ corresponding to a maximum relative bandwidth $\hat{\mu}$, and one H user transmits using the remaining bandwidth at $R(\gamma_{\mathrm{S}}, 1-\hat{\mu})$. As discussed we may want to have more L users transmit in OFDMA-1 depending on the SNR and relative L rate. In general, the "corner points" mentioned above are those for which $(U-1)$ L users transmit at their maximum speed and another user transmits in the remaining band with $R(\gamma_{\mathrm{S}}, 1-(U-1)\hat{\mu})$. If this latter rate is larger than $\hat{r}_L$ this user must be an H user. Otherwise it may also be an L user.

It can be shown that if $R(\gamma_{\mathrm{S}}, \mu)$ is convex in μ – this is the case for our numerical example – then for all ρ selecting the same two modes maximizes the sum rate. These are mode A as above and mode B with U_B L users only, i. e., $\rho_B = 1$. Of these, $U_B - 1$ transmit at their maximum rate $\hat{r}_L$ with relative bandwidth $\hat{\mu}$ and one may transmit at a lower rate in a smaller bandwidth such that the entire band is used. U_B depends on $\hat{\mu}$ which, in turn, depends implicitly on the SNR γ_{S}. We have

$$U_B = \lceil \hat{\mu}^{-1} \rceil \quad \text{with} \quad R(\gamma_{\mathrm{S}}, \hat{\mu}) = \hat{r}_L, \tag{16}$$

where $\lceil \cdot \rceil$ denotes the ceiling operation. For the sum rate it follows from (15) that

$$R_S^{AB}(\rho) = \left(\frac{\rho}{R_S(\rho_B)} + \frac{1-\rho}{R_S(\rho_A)} \right)^{-1}. \tag{17}$$

Figs. 1, 2 also contain the curves for OFDMA-AB (triangles). In Fig. 1 the different solid and dotted sections of the curve correspond to U_B varying from 1 to 9 according to increasing γ_{S}. For very low SNR $U_B = 1$ such that the scheme is effectively TDMA. With higher SNR, initially, the sum rate is slightly lower than for TDMA. As for single-mode OFDMA this is due to a loss in diversity per user. However, the rate loss with OFDMA-AB is less dramatic and within a smaller SNR region than with OFDMA-1. OFDMA-AB throughput is the highest among the schemes. Note that in the large SNR regime the two OFDMA variants tend to perform equally well – for rather different numbers of simultaneously active users. In Fig. 2 we see that this is true for the entire range of relative L rates.

VI. Conclusion

We have investigated the achievable sum rate of TDMA and OFDMA under system energy and rate constraints. The constraints on relative L rate and maximum requested rates have been obtained from an abstraction of measured packet size distributions. We put particular emphasis on WLAN systems as in [4]. For these we found that the need of every such system to transmit a large number of small packets severely limits the throughput of conventional time-multiplexing based multiple access if the number of bits that must be transmitted in one OFDM symbol gets large. On the other hand, with otherwise unchanged parameters, OFDMA reduces the number of bits that one user must transmit in one OFDM symbol and the throughput is much increased over TDMA.

Acknowledgment

This research was supported by the German Ministry of Education and Research within the project *Wireless Gigabit with Advanced Multimedia Support* (WIGWAM) under grant 01 BU 370.

References

[1] E. Biglieri, J. Proakis, and S. Shamai (Shitz), "Fading channels: Information-theoretic and communications aspects," *IEEE Trans. Inform. Theory*, vol. 44, no. 6, pp. 2619–2692, Oct. 1998.

[2] A. Goldsmith, *Wireless Communications*. New York: Cambridge Univ. Press, 2005.

[3] H. Boche and M. Wiczanowski, "Queueing theoretic optimal scheduling for multiple input multiple output multiple access channel," in *Proc. ISSPIT 2003*. IEEE, 2003, pp. 576–579.

[4] G. Fettweis and R. Irmer, "WIGWAM: System concept development for 1 Gbit/s air interface," in *WWRF 14*, San Diego, USA, July 2005.

[5] D. Tang and M. Baker, "Analysis of a local-area wireless network," in *Proc. 6th ACM MOBICOM*. ACM Press, 2000, pp. 1–10.

[6] D. Kotz and K. Essien, "Analysis of a campus-wide wireless network," in *Proc. 8th ACM MOBICOM*. ACM Press, 2002, pp. 107–118.

[7] C. Na, J. K. Chen, and T. S. Rappaport, "Hotspot traffic statistics and throughput models for several applications," in *Proc. GLOBECOM 2004*, vol. 5. IEEE, Nov. 2004, pp. 3257–3263.

[8] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc. Intern. Conf. Commun., ICC 1995*, vol. 1. IEEE, June 1995, pp. 331–335.

[9] G. Caire, G. Taricco, and E. Biglieri, "Optimum power control over fading channels," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1468–1489, July 1999.

[10] T. Deckert and G. Fettweis, "Achievable rates of asymmetrical multiple access in block-fading channels," in *Proc. ICC 2006*. IEEE, 2006.