

# IQ Imbalance Compensation for Receivers with 1-Bit Quantization and Oversampling

Konstantin Kochs, Florian Mann, Meik Dörpinghaus, and Gerhard Fettweis

Vodafone Chair Mobile Communications Systems, Technische Universität Dresden, Germany

{konstantin.kochs, florian.mann, meik.doerpinghaus, gerhard.fettweis}@tu-dresden.de

**Abstract**—Leveraging broad frequency ranges, as they are available in the *terahertz* (THz) regime, for high-bandwidth communications makes the energy consumption of the receiver's *analog-to-digital converter* (ADC) a major challenge. To solve this problem, a promising approach is the use of ADCs with 1-bit amplitude resolution in combination with temporal oversampling. This coarse quantization, however, necessitates the redesign of most receiver algorithms. This work addresses the estimation and compensation of *in-phase and quadrature* (IQ) imbalance, a critical factor regarding the communication performance. It is shown that amplitude imbalance has no deteriorating effect on the communication performance, while the impact of phase imbalance is significant. To address this, an efficient unbiased estimator for IQ phase imbalance based on 1-bit quantized observations is derived, achieving the *Cramér-Rao lower bound* (CRLB). Its accuracy, its robustness against inaccurate timing conditions, and its performance gains in case of oversampling are assessed. Furthermore, a guideline is derived, which either recommends or discourages compensation with regard to the expected variance of an individual estimate. Thereafter, this work discusses approaches to carry out the compensation of the estimated IQ imbalance. Eventually, simulations reveal the estimator's ability to significantly enhance communication performance.

## I. INTRODUCTION

Future communications systems are expected to employ increasingly high bandwidths. The decades-long survey [1] suggests an empirical lower bound on the *analog-to-digital converter's* (ADC's) energy consumption per conversion step, which scales linearly with a sampling frequency above approximately 1 GHz—a range typically required for *terahertz* (THz) communications. Consequently, with such sampling frequencies the power draw increases quadratically. The ADCs thus become one of the major power sinks in the circuitry of receivers. In relation with its amplitude resolution, however, the ADC's power consumption grows exponentially [2]. To cope with this behavior, the amplitude resolution can be ultimately reduced to one bit, which is used for modulation schemes like *zero-crossing modulation* (ZXM) [3]. Further improvements in energy efficiency and chip area are due to such 1-bit ADCs' typically less complex hardware implementations. Moreover, the use of 1-bit ADCs significantly relaxes the requirements on automatic gain control. Employing such ADCs, however,

necessitates a reconsideration of the baseband design in comparison to conventional communication systems. In this regard there is already significant prior work on timing offset estimation and synchronization [4], [5], carrier frequency offset estimation [6], phase offset estimation [4], [7], and phase noise tracking [8]. Another essential task, which is yet to be solved for receivers with 1-bit quantization, is the estimation and compensation of IQ imbalance—an impairment to the orthogonality of the *in-phase* (I) and the *quadrature* (Q) signal components. In THz communications this becomes a major performance-limiting factor [9], [10].

In [11], [12] a lower bound on the *Fisher information* (FI) for generic channel parameter estimation is derived; an upper bound on the FI for timing, frequency, and phase estimation is found in [13]. Furthermore, [14]–[16] describe specific estimation approaches for such generic parameters. In [17] the estimation of a channel's impulse response for several ADC resolutions is considered, including 1 bit.

All this prior work assumes circularly symmetric complex noise at the input of the ADC. Impaired by IQ imbalance, however, the signal to be quantized does not fulfill this assumption; the I component and the Q component of the additive noise are correlated.

IQ imbalance estimation without consideration of the quantizer resolution is a well-known topic in literature, employing models for IQ phase and amplitude imbalance, which are not necessarily equivalent to each other. They differ, for example, in the assumptions about frequency dependency, e.g., [10], [18], [19]. Some assume only one axis, either the I or the Q axis, to be rotated creating phase imbalance as in [10], [18], [20]–[22], others assume both axes to be rotated symmetrically against each other by the same angle as in [23], [24], and others model their rotation independently, e.g., [19]. Moreover, some works as [25] assume the transmit signal to be known, other works as [20] consider the signal phase to be unknown or even devote themselves to blind estimation as considered in [24], [26].

In relation to the plenty of literature about IQ imbalance, research about typical magnitudes of IQ phase imbalance appears to be scarce. Surveying the exemplary IQ phase imbalances brought up in several works [22]–[28] suggest that the IQ phase imbalance is to be expected within the range of  $1^\circ$  to  $10^\circ$ . In THz communication, however, [9] and [10] observe a tendency for even higher imbalances.

This work aims to investigate the impact of IQ imbalance on the attainable performance of communication systems employing receivers with 1-bit ADCs and temporal oversampling.

This work is part of the TeraGreen project which received funding from the Smart Networks and Services Joint Undertaking (SNS JU) under the European Union's (EU's) Horizon Europe research and innovation programme under Grant Agreement No. 101139117. Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the EU or the SNS JU. Neither the EU nor the granting authority can be held responsible for them.

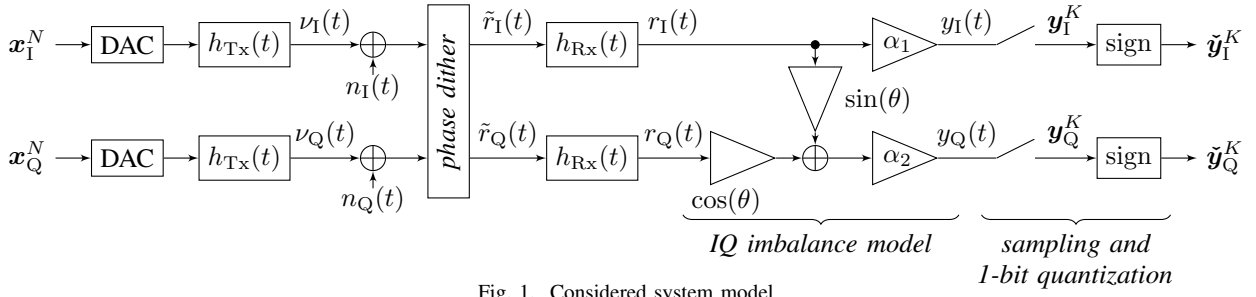


Fig. 1. Considered system model

A way to efficiently estimate and compensate the IQ phase imbalance is proposed, and the robustness against inaccurate timing conditions and benefit from oversampling of the derived algorithm are assessed. Moreover, it is discussed where the IQ imbalance estimation and compensation should be located within the receiver synchronization chain.

The remainder of this paper is structured as follows. The models for the signaling scheme, the baseband channel, and IQ imbalance are introduced in Section II. Based on this, Section III is devoted to the derivation of the Fisher information and an efficient estimator for IQ phase imbalance as well as a measure of accuracy for the IQ phase imbalance estimation. Thereafter, in Section IV ways to realize IQ imbalance compensation are described and evaluated with regard to the estimation error propagation. Finally, the communication performance in the presence of IQ imbalance, the improvements due to compensation, and the influence of sample time offsets and receiver-side oversampling on the estimation performance are assessed in Section V. The main results and achievements of this work are summarized in Section VI.

Throughout this work, boldface letters denote multivariate parameters. Matrices are described by uppercase and vectors by lowercase letters. For complex signals (e.g.,  $\mathbf{x}$ ),  $x_I$  and  $x_Q$  signify their I and Q components. For the sake of conciseness when working with probability distributions, complex-valued entities are written as bivariate vectors, e.g.,  $\mathbf{x} = [x_I, x_Q]^T$ . Their magnitude and argument are described by  $|\cdot|$  and  $\angle\{\cdot\}$ . Random variables are denoted by upright sans-serif letters (e.g.,  $\mathbf{x}_I$ ) as opposed to their realizations (e.g.,  $x_I$ ). Sequences are described by their length given as a superscript, while a single element of a sequence is described by the subscript index, e.g.,  $\mathbf{x}^N = [x_0, \dots, x_{N-1}]$ .

## II. SYSTEM MODEL

The considered system model as subsequently described is displayed in Fig. 1.

### A. Transmit Signal and Channel Model

The employed signaling scheme is based on *quadrature phase-shift keying* (QPSK) symbols. That is, a transmit symbol sequence

$$\mathbf{x}^N = [\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{N-1}] \quad (1)$$

is composed of symbols  $\mathbf{x}_i \in \{[p, q]^T : p, q \in \{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\}\}$ . The transmit signal  $\mathbf{v}(t)$  is thus constructed with the transmit filter  $h_{TX}(t)$  as

$$\mathbf{v}(t) = \sum_{i=0}^{N-1} \mathbf{x}_i h_{TX}(t - iT_{TX}). \quad (2)$$

$T_{TX}$  signifies the Nyquist interval corresponding to  $h_{TX}(t)$ . Hence, the symbol rate is  $1/T_{TX}$ . When transmitted over the channel,  $\mathbf{v}(t)$  is impaired by a complex and circularly symmetric *additive white Gaussian noise* (AWGN) process  $\mathbf{n}(t)$  with one-sided power spectral density  $N_0$ . With

$$\mathcal{P}_s = \lim_{T_s \rightarrow \infty} \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} |\mathbf{v}(t)|^2 dt \quad (3)$$

the transmit signal power, the *signal-to-noise ratio* (SNR) is defined as  $\text{SNR} = \mathcal{P}_s T_{TX} / N_0$ .

We assume perfect timing and carrier frequency synchronization of the receiver. Since receiver-side phase synchronization, however, is carried out *after* the compensation of IQ imbalance, the received signal has an arbitrary phase offset. Yet, the performance of channel parameter estimation depends on the signal phase [13]. To remove the dependency on the unknown signal phase, a random uniform phase dither is employed. That is, the received signal is artificially distorted by uniform and uncorrelated phase noise  $\phi_d \sim \mathcal{U}(0, 2\pi)$  which is drawn independently for every temporal sample. In practical systems this can be realized as described in [13] by down-converting the passband signal to a low *intermediate frequency* (IF)  $f_{IF}$ , which multiplied by the sample period  $T$  (i.e.,  $f_{IF}T$ ) is an irrational number. Due to phase dithering the phase offset at the receiver becomes irrelevant and is therefore ignored in the employed model. At the receiver, the phase dithered signal is composed as

$$\tilde{\mathbf{r}}(t, \phi_d) = \tilde{\mathbf{v}}(t, \phi_d) + \mathbf{n}(t). \quad (4)$$

In this model, the stochastic properties of the noise process  $\mathbf{n}(t)$  are unaffected by the phase dither owing to its circularly symmetric distribution.

Subsequently, the signal is filtered with the receive filter  $h_{RX}(t)$  having Nyquist period  $T_{RX}$  and energy  $1/T_{RX}$ , i.e.,

$$\mathbf{r}(t, \phi_d) = (\tilde{\mathbf{v}} * h_{RX})(t, \phi_d) + (\mathbf{n} * h_{RX})(t) \quad (5)$$

$$= \mathbf{s}(t, \phi_d) + \mathbf{m}(t), \quad (6)$$

where the convolution  $*$  is carried out element-wise. The variance of the filtered noise process  $\mathbf{m}(t)$  is  $\sigma_m^2 = N_0/T_{RX}$ .

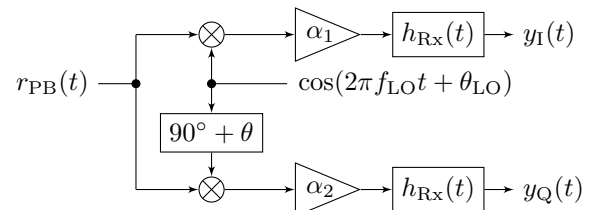


Fig. 2. Down-conversion stage including local oscillator

## B. IQ Imbalance Model

As defined, e.g., in [25], the down-conversion of a passband signal  $r_{PB}(t)$  at the receiver can be modeled as shown in Fig. 2. The *local oscillator* (LO), set to frequency  $f_{LO}$ , has a phase offset  $\theta_{LO}$ —a common phase error on both I and Q, and thus a contribution to the receive signal's phase offset, which has no impact due to the employed phase dithering.

To down-convert the Q component, the signal generated by the LO ought to be shifted by exactly  $90^\circ$ , resulting in a sine signal. The phase shifter, however, is inaccurate by a phase offset  $\theta$ . This impairment only affects the Q component and therefore impacts the IQ phase imbalance. In addition, the gain factors  $\alpha_1$  and  $\alpha_2$  effect IQ amplitude imbalance.

Thus the equivalent baseband model for the IQ imbalance is described by the fixed but unknown parameters  $\theta$  and  $\alpha = [\alpha_1, \alpha_2]^T$  for IQ phase and amplitude imbalance, where  $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$  and  $\alpha_1, \alpha_2 > 0$ . The IQ imbalance matrix is

$$\Psi(\alpha, \theta) = \begin{bmatrix} \alpha_1 & 0 \\ \alpha_2 \sin(\theta) & \alpha_2 \cos(\theta) \end{bmatrix}, \quad (7)$$

composing the received signal *with* IQ imbalance as

$$\mathbf{y}(t) = \Psi(\alpha, \theta) \mathbf{r}(t, \phi_d). \quad (8)$$

## C. Sampling and 1-Bit Quantization

Many works, e.g., [3], [13], have shown receiver-side oversampling by a factor of  $M_{Rx} > 1$  to be beneficial for parameter estimation. Thus, sampling with a period of  $T = \frac{T_{Rx}}{M_{Rx}}$ , the  $k$ -th element of a sampled sequence  $\mathbf{y}^K$  of the received signal  $\mathbf{y}(t)$  is  $\mathbf{y}_k = \mathbf{y}(kT) = [y_{I,k}, y_{Q,k}]^T$  for  $k \in \{0, 1, \dots, K-1\}$ . Respectively, the sampled signal and noise components are  $\mathbf{s}_k(\phi_d) = \mathbf{s}(kT, \phi_d)$  and  $\mathbf{m}_k = \mathbf{m}(kT)$ . For all the analytical evaluations in this work,  $h_{Rx}$  is assumed to be an ideal low-pass filter, and sampling is assumed to occur at the Nyquist rate  $1/T_{Rx}$ . Consequently,  $M_{Rx} = 1$  and the noise samples in  $\mathbf{m}^K$  are *independent and identically distributed* (i.i.d.) with variance  $\sigma_m^2$ .

The received and sampled signal  $\mathbf{y}^K$  is thereafter quantized with 1-bit resolution for the I as well as the Q component, i.e.,

$$\tilde{\mathbf{y}}_k = \text{vsign}(\mathbf{y}_k), \quad (9)$$

where the *vsign* function applies the sign function to both vector elements yielding the quantized sequence of observations  $\tilde{\mathbf{y}}^K$ . The sign function is defined as  $\text{sign}(\beta) = 1$  if  $\beta \geq 0$ , and  $\text{sign}(\beta) = -1$  otherwise.

## III. ESTIMATION OF IQ IMBALANCE

### A. Likelihood Function

This section aims to derive  $P_{\tilde{\mathbf{y}}^K}(\tilde{\mathbf{y}}^K; \alpha, \theta)$ , that is, the likelihood of an observed sequence  $\tilde{\mathbf{y}}^K$  for a fixed phase imbalance  $\theta$  and an amplitude imbalance  $\alpha$ .

1) *Likelihood of a Single Observation*: As a starting point, given a dither phase realization  $\phi_d$  and knowing  $\mathbf{x}^N$ , which determine  $\mathbf{s}_k(\phi_d)$ , the distribution of one unquantized observation  $\mathbf{y}_k$  follows directly from (4), (6) and (8) as

$$\mathbf{y}_k \sim \mathcal{N}(\Psi(\alpha, \theta) \mathbf{s}_k(\phi_d), \mathbf{R}) \quad (10)$$

with the covariance matrix

$$\mathbf{R} = \sigma_m^2 \Psi(\alpha, \theta) \Psi(\alpha, \theta)^T. \quad (11)$$

The corresponding likelihood function is

$$P_{\mathbf{y}_k|\phi_d, \mathbf{x}^N}(\mathbf{y}_k|\phi_d, \mathbf{x}^N; \alpha, \theta) = \frac{1}{2\pi\sqrt{\det \mathbf{R}}} \times e^{\left(-\frac{1}{2\sigma_m^2}(\Psi^{-1}(\alpha, \theta)\mathbf{y}_k - \mathbf{s}_k(\phi_d))^T(\Psi^{-1}(\alpha, \theta)\mathbf{y}_k - \mathbf{s}_k(\phi_d))\right)}, \quad (12)$$

wherein  $\sqrt{\det \mathbf{R}} = \sigma_m^2 \alpha_1 \alpha_2 \cos(\theta)$ .

The likelihood function after 1-bit quantization, see (9), is obtained by integrating  $P_{\mathbf{y}_k|\phi_d, \mathbf{x}^N}(\mathbf{y}_k|\phi_d, \mathbf{x}^N; \alpha, \theta)$  over all  $\mathbf{y}_k$  within the quadrant  $V(\tilde{\mathbf{y}}_k) = \{\mathbf{y}_k : \text{vsign}(\mathbf{y}_k) = \tilde{\mathbf{y}}_k\}$  in the IQ space corresponding to a quantized observation  $\tilde{\mathbf{y}}_k$ , i.e.,

$$P_{\tilde{\mathbf{y}}_k|\phi_d, \mathbf{x}^N}(\tilde{\mathbf{y}}_k|\phi_d, \mathbf{x}^N; \alpha, \theta) = \iint_{V(\tilde{\mathbf{y}}_k)} P_{\mathbf{y}_k|\phi_d, \mathbf{x}^N}(\mathbf{y}_k|\phi_d, \mathbf{x}^N; \alpha, \theta) d\mathbf{y}_k. \quad (13)$$

Resolving the condition on  $\phi_d$ ,

$$P_{\tilde{\mathbf{y}}_k|\mathbf{x}^N}(\tilde{\mathbf{y}}_k|\mathbf{x}^N; \alpha, \theta) = \frac{1}{2\pi} \int_0^{2\pi} P_{\tilde{\mathbf{y}}_k|\phi_d, \mathbf{x}^N}(\tilde{\mathbf{y}}_k|\phi_d, \mathbf{x}^N; \alpha, \theta) d\phi_d, \quad (14)$$

which by combining (12) to (14) equals

$$P_{\tilde{\mathbf{y}}_k|\mathbf{x}^N}(\tilde{\mathbf{y}}_k|\mathbf{x}^N; \alpha, \theta) = P_{\tilde{\mathbf{y}}_k}(\tilde{\mathbf{y}}_k; \theta) = \frac{1}{4} + \frac{\tilde{y}_{I,k}\tilde{y}_{Q,k}}{2\pi}\theta. \quad (15)$$

The proof of (15), which relies on transforming  $\mathbf{y}_k$  in (13) into polar coordinates, is omitted for brevity. The result is intuitively consistent if  $\frac{\theta}{2\pi}$  is interpreted as a fraction of the unit circle. Considering (15), the quantized observations  $\tilde{\mathbf{y}}^K$  turn out to be independent of the transmit signal  $\mathbf{v}$ , the amplitude imbalance  $\alpha$ , and even the noise variance  $\sigma_m^2$ . An intuitive explanation for this is that the received signal's phase becomes negligible, overruled by the phase dithering, and the amplitude information is lost through 1-bit quantization.

2) *Likelihood of Multiple Observations*: Since the observations in a sequence  $\tilde{\mathbf{y}}^K$  are i.i.d. and independent of  $\mathbf{x}^N$ , the joint likelihood is

$$P_{\tilde{\mathbf{y}}^K}(\tilde{\mathbf{y}}^K; \theta) = \prod_{k=0}^{K-1} P_{\tilde{\mathbf{y}}_k}(\tilde{\mathbf{y}}_k; \theta) \quad (16)$$

$$= \left(\frac{1}{4} + \frac{\theta}{2\pi}\right)^{g_{\tilde{\mathbf{y}}}} \left(\frac{1}{4} - \frac{\theta}{2\pi}\right)^{K-g_{\tilde{\mathbf{y}}}}, \quad (17)$$

wherein  $g_{\tilde{\mathbf{y}}} = g(\tilde{\mathbf{y}}^K) \in [0, K]$  describes the amount of observations made in the quadrants I and III, where  $\tilde{y}_{I,k}\tilde{y}_{Q,k} = 1$ , i.e.,

$$g(\tilde{\mathbf{y}}^K) = \frac{K}{2} + \frac{1}{2} \sum_{k=0}^{K-1} \tilde{y}_{I,k}\tilde{y}_{Q,k}. \quad (18)$$

### B. Fisher Information for IQ Phase Imbalance

Before solving the estimation problem, it is helpful to assess the information about  $\theta$  contained in a sequence of observations, namely, the FI

$$F_{\tilde{\mathbf{y}}^K}(\theta) = -\mathbb{E} \left[ \frac{\partial^2}{\partial \theta^2} \ln P_{\tilde{\mathbf{y}}^K}(\tilde{\mathbf{y}}^K; \theta) \right]. \quad (19)$$

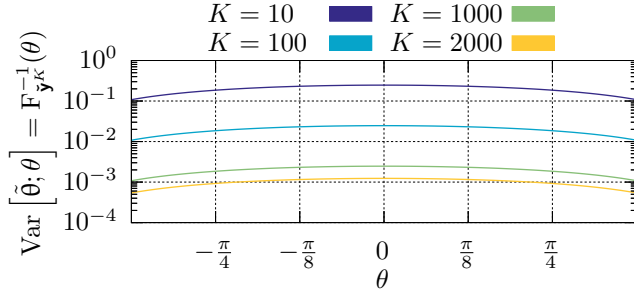


Fig. 3. Variance of MVUE error  $\tilde{\theta} = \hat{\theta} - \theta$ , i.e., the CRLB

Using (17) the first and second derivative of the *log-likelihood function* (LLF) are given by

$$\frac{\partial}{\partial \theta} \ln P_{\tilde{\mathbf{y}}^K}(\tilde{\mathbf{y}}^K; \theta) = \frac{g(\tilde{\mathbf{y}}^K) \pi - (\frac{\pi}{2} + \theta) K}{\frac{\pi^2}{4} - \theta^2} \quad \text{and} \quad (20)$$

$$\frac{\partial^2}{\partial \theta^2} \ln P_{\tilde{\mathbf{y}}^K}(\tilde{\mathbf{y}}^K; \theta) = \frac{2\pi\theta}{(\frac{\pi^2}{4} - \theta^2)^2} g(\tilde{\mathbf{y}}^K) - \frac{K}{(\frac{\pi}{2} - \theta)^2}. \quad (21)$$

Furthermore, using (18),

$$\mathbb{E} \left[ g(\tilde{\mathbf{y}}^K) \right] = \frac{K}{2} + \frac{K}{\pi} \theta = K \left( \frac{1}{2} + \frac{\theta}{\pi} \right). \quad (22)$$

Combining (19), (21) and (22) yields the FI in closed form as

$$F_{\tilde{\mathbf{y}}^K}(\theta) = \frac{K}{\frac{\pi^2}{4} - \theta^2}. \quad (23)$$

### C. Minimum Variance Unbiased Estimator

The reciprocal of the FI, the *Cramér-Rao lower bound* (CRLB), represents a lower bound on the error variance of any unbiased estimator for  $\theta$ . As stated in [29, Theorem 3.1], there is an efficient estimator—i.e., an estimator  $\hat{\theta}$  that attains the CRLB—iff there exists a function  $\xi(\tilde{\mathbf{y}}^K)$ , which is independent of  $\theta$  and for which

$$\frac{\partial}{\partial \theta} \ln P_{\tilde{\mathbf{y}}^K}(\tilde{\mathbf{y}}^K; \theta) = F_{\tilde{\mathbf{y}}^K}(\theta) \left( \xi(\tilde{\mathbf{y}}^K) - \theta \right) \quad (24)$$

holds. Using (20) and (23) this condition is found to hold and the efficient *minimum variance unbiased estimator* (MVUE) is determined as

$$\hat{\theta} = \xi(\tilde{\mathbf{y}}^K) = g(\tilde{\mathbf{y}}^K) \frac{\pi}{K} - \frac{\pi}{2} = \frac{\pi}{2K} \sum_{k=0}^{K-1} \tilde{y}_{I,k} \tilde{y}_{Q,k}. \quad (25)$$

It is linear with the observations  $\tilde{y}_{I,k}$  and  $\tilde{y}_{Q,k}$ , of linear complexity scaling with the number of observations, and also discrete-valued—properties facilitating simple and accurate hardware implementation.

Since the MVUE is efficient, the variance of the estimation error  $\tilde{\theta} = \hat{\theta} - \theta$  is the CRLB, i.e.,  $\text{Var}[\tilde{\theta}; \theta] = F_{\tilde{\mathbf{y}}^K}^{-1}(\theta)$ . Fig. 3 displays  $\text{Var}[\tilde{\theta}; \theta]$  as a function of the IQ phase imbalance for different observations length  $K$ .

### D. Profitability of Estimates

If the estimation error variance is too high compared to the IQ imbalance that is to be compensated—i.e., if  $\text{Var}[\tilde{\theta}; \theta] \geq \theta^2$ —, it is advisable to refrain from compensation. Obviously, this decision is impossible, since the actual IQ

imbalance parameter  $\theta$  is unknown. As the probability of the sufficient statistic  $g$  for i.i.d. observations  $\tilde{\mathbf{y}}^K$  equals

$$P_{g(\tilde{\mathbf{y}}^K)} \left( g(\tilde{\mathbf{y}}^K); \theta \right) = \binom{K}{g(\tilde{\mathbf{y}}^K)} P_{\tilde{\mathbf{y}}^K}(\tilde{\mathbf{y}}^K; \theta), \quad (26)$$

where  $\binom{p}{q}$  denotes the binomial coefficient, however, the posterior probability can be inferred as

$$P_{\theta|g(\tilde{\mathbf{y}}^K)} \left( \theta | g(\tilde{\mathbf{y}}^K) \right) = \frac{P_{g(\tilde{\mathbf{y}}^K)} \left( g(\tilde{\mathbf{y}}^K); \theta \right)}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} P_{g(\tilde{\mathbf{y}}^K)} \left( g(\tilde{\mathbf{y}}^K); \theta' \right) d\theta'}. \quad (27)$$

Thus the certainty of an estimate can be assessed by the expected variance of  $\theta$  given  $g(\tilde{\mathbf{y}}^K)$ , or equivalently, given  $\hat{\theta}$ . Hence, the condition that the expected estimation error variance is smaller than the expected quadratic error made by not compensating by the estimate is found as

$$\mathbb{E}_{\theta|g(\tilde{\mathbf{y}}^K)} \left[ \text{Var} \left( \tilde{\theta} | g(\tilde{\mathbf{y}}^K) \right) \right] < \mathbb{E}_{\theta|g(\tilde{\mathbf{y}}^K)} \left[ \theta^2 \right], \quad (28)$$

which can be reduced to

$$|\hat{\theta}| > \frac{\pi}{K} \sqrt{\frac{K+2}{2K+2}}. \quad (29)$$

Only if (29) holds, the estimate  $\hat{\theta}$  should be deemed accurate enough to motivate beneficial compensation of  $\theta$ .

## IV. COMPENSATION OF IQ IMBALANCE

In case the IQ imbalance compensation should be carried out in the digital domain it would perhaps be best to consider this in conjunction with maximum a posteriori decoding for a specific coding scheme, which exceeds the scope of this work. Nevertheless, there are several ways to compensate the IQ phase imbalance in the analog domain.

### A. Correction of Shifter Inside Mixer

In theory, an accurate approach to correct the IQ phase imbalance is to calibrate the impaired phase shifter. With an estimate  $\hat{\theta} = \theta + \tilde{\theta}$ , this would leave a residual IQ phase imbalance of  $\theta_c = -\tilde{\theta}$ , i.e., the negative estimation error. Naturally, this approach requires the phase shifter to be adjustable with sufficient accuracy.

### B. Inversion of the Imbalance Matrix

Another, more practical approach has already been applied, for instance, in [18], [21], [28]. By assuming the IQ amplitude imbalance to be moderate it might be sufficient to approximate it by  $\alpha = [1, 1]^T$ . Hence, the idea of this approach is to multiply the analog signal  $\mathbf{y}(t)$  (or equivalently  $\mathbf{y}^K$  after sampling) by  $\hat{\Psi}^{-1} = \Psi^{-1}([1, 1]^T, \hat{\theta})$ . Utilizing that the afterwards carried out 1-bit quantization is oblivious of the actual amplitude of the signal components, the quantized signal is equivalently given by

$$\text{vsign} \left( \hat{\Psi}^{-1} \mathbf{y}(t) \right) = \text{vsign} \left( \begin{bmatrix} 1 & 0 \\ -\sin(\hat{\theta}) & 1 \end{bmatrix} \mathbf{y}(t) \right). \quad (30)$$

The corrected signal  $\mathbf{y}_c(t)$  is thus given by

$$\mathbf{y}_c(t) = \left[ y_I(t), y_Q(t) - \sin(\hat{\theta}) y_I(t) \right]^T. \quad (31)$$

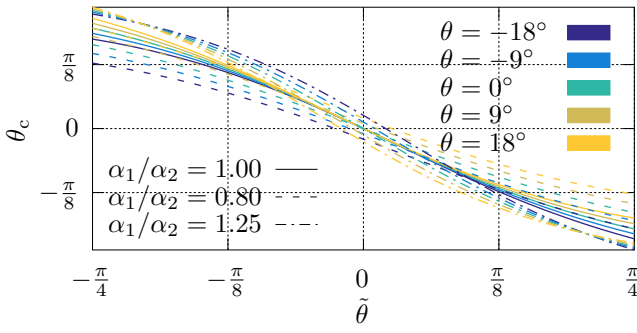


Fig. 4. Residual IQ phase imbalance angle  $\theta_c$  after estimating  $\theta$  using the MVUE with an error  $\tilde{\theta}$  and compensating it by inverting the imbalance matrix

In a practical implementation, the approximation  $\sin(\hat{\theta}) \approx \hat{\theta}$  can be employed to potentially reduce hardware complexity. Compensating with an estimate  $\hat{\theta} = \theta + \tilde{\theta}$ , the residual IQ phase imbalance  $\theta_c$  after compensation using (31) is found by multiplying  $\hat{\Psi}^{-1}\Psi(\alpha, \theta)$ , resulting in

$$\theta_c = \arctan\left(\tan(\theta) - \frac{\alpha_1 \sin(\hat{\theta})}{\alpha_2 \cos(\theta)}\right). \quad (32)$$

For several phase and amplitude imbalances Fig. 4 shows  $\theta_c$  in relation to the estimation error  $\tilde{\theta}$ . Without amplitude imbalance (i.e.,  $\alpha_1/\alpha_2 = 1$ ), the remaining IQ phase imbalance is approximately the negative estimation error, that is  $\theta_c \approx -\tilde{\theta}$ . Thus, this compensation approach approximates the behavior of the approach discussed in Section IV-A. It appears that if  $\alpha_1$  is moderately close to  $\alpha_2$ , this approximation still holds. This approach poses an alternative to the one described in Section IV-A without necessitating an adjustable phase shifter.

## V. NUMERICAL EVALUATION

For the following numerical evaluations, and contrary to previous assumptions, both  $h_{Tx}(t)$  and  $h_{Rx}(t)$  are root-raised cosine filters with a roll-off factor of 0.6.

### A. Bit Error Rate

As a measure of communication performance, the uncoded *bit error rate* (BER) is empirically determined by comparing the sign errors between a large number of transmit symbol sequences  $\mathbf{x}^N$  and the corresponding observation sequences  $\tilde{\mathbf{y}}^K$ . For this evaluation, we do not consider any oversampling, i.e.,  $M_{Rx} = 1$  and therefore  $N = K$ . Furthermore, we consider the IQ imbalance compensation by the MVUE  $\hat{\theta}$  to be carried out perfectly in the analog domain as described in Section IV-A. Fig. 5 (a) shows that the uncoded BER substantially increases with IQ phase imbalance. The BERs after compensation are shown in Fig. 5 (b) and (c). Since the estimation error of the MVUE is relatively flat within the range of the chosen values for  $\theta$ , cf. Fig. 3, the resulting BERs after compensation are close to one another, as displayed in Fig. 5 (b) and (c). Comparison between Fig. 5 (b) and (c) furthermore demonstrates the impact of the estimator variance on the BER, depending on the number of observations  $K$ . For a small number of observations (e.g.,  $K = 100$ ), compensation can even increase the residual IQ imbalance, thereby raising the BER. With a sufficient number of observations (e.g.,  $K = 1000$ ), communication performance as without IQ imbalance can be nearly restored.

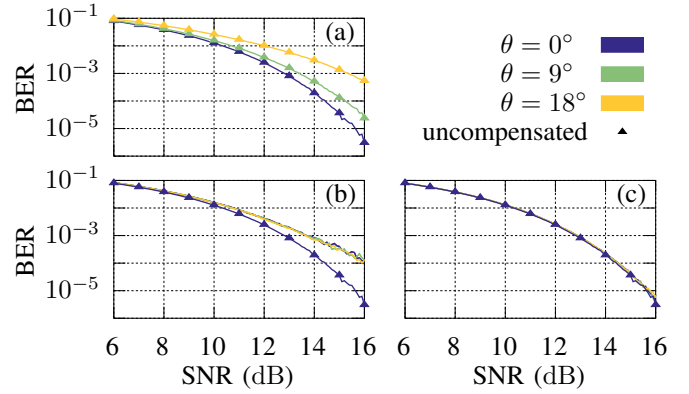


Fig. 5. Uncoded BER for several values of the IQ imbalance (a) before compensation, after compensation by the MVUE with (b)  $K = 100$ , and (c)  $K = 1000$  observations

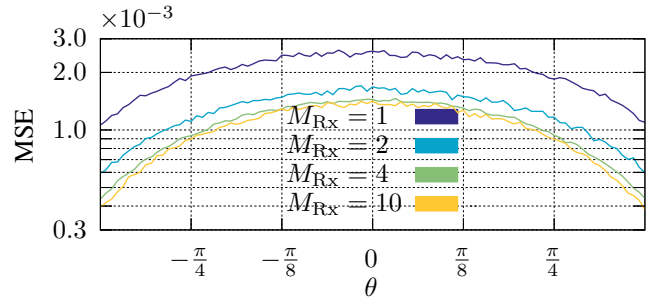


Fig. 6. MSE of the MVUE with  $K = 1000M_{Rx}$  observations, while oversampling by factor  $M_{Rx}$

### B. Timing Independence

If timing synchronization is inaccurate or carried out after IQ imbalance estimation, the samples are time-shifted by an offset  $\Delta t$ , i.e.,  $\tilde{\mathbf{y}}_k = \text{vsign}(\mathbf{y}(kT + \Delta t))$ . Fortunately, as has been derived in Sections III-A1, III-B and III-C, the MVUE performance is independent of the transmit signal  $\mathbf{v}$ , and therefore unaffected by non-optimal timing conditions. This is supported by numerical results obtained by simulation, which are not shown here. These results highlight that the IQ phase imbalance estimation is independent of the receiver timing synchronization stage, which can therefore be performed later in the signal processing chain, potentially benefiting from the already compensated IQ imbalance.

### C. Influence of Oversampling

As the recorded noise samples  $\mathbf{m}^K$  are correlated when using an oversampling factor  $M_{Rx} > 1$ , the employed estimator  $\hat{\theta}$  is mismatched in this context. When the observation time is held constant, such that  $K \propto M_{Rx}$ , oversampling improves the estimation performance, see Fig. 6. Furthermore, the MSE reduction achieved by increasing  $M_{Rx}$  is significantly larger for low oversampling rates than for high ones. Therefore, it is presumably inadvisable to oversample with more than  $M_{Rx} = 4$  solely for the sake of IQ imbalance compensation, considering the increased computational effort and energy consumption. Note that the observed performance gain with oversampling only results from the increased number of observations  $K$ . Further simulations revealed that, in case  $K$  is fixed, increasing  $M_{Rx}$  leads to a performance degradation.

This results from the mismatch of the estimator, which ignores the temporal sample correlation. The benefit of an increased number of observations outweighs the performance loss by sample correlation, cf. Fig. 6. For larger  $M_{\text{Rx}}$ , these opposing effects appear to offset each other.

## VI. CONCLUSION

This work assessed the impact of IQ imbalance on a receiver with 1-bit quantization employing oversampling. Since 1-bit quantization resolves only the sign of the received signal, IQ amplitude imbalance has no effect on the quantized observations. Therefore, no amplitude imbalance compensation is required. For phase imbalance, in the present work we derived an estimator in closed form and proved its optimality in the case of Nyquist-rate sampling. The proposed estimator is efficient, robust against inaccurate timing conditions, independent of the receiver phase offset due to phase dithering, and independent of the SNR. Its concise algebraic form enables an exact and straightforward implementation in digital circuitry. The estimator's independence of the transmit signal further implies that IQ imbalance estimation can be carried out before receiver-side timing and phase synchronization, which may consequently benefit from the reduced IQ imbalance. In addition, two alternative compensation approaches in the analog domain have been compared with regard to the propagation of estimation errors.

Subsequent studies could attempt to derive an IQ imbalance estimator considering oversampling and compare with the estimator derived in this work.

Overall, the results of this work provide an effective approach to mitigate IQ imbalance in receivers with 1-bit quantization, thereby significantly enhancing communication performance and contributing to the advancement of energy-efficient THz communication.

## REFERENCES

- [1] B. Murmann, "ADC Performance Survey 1997-2024," [Online]. Available: <https://github.com/bmurmann/ADC-survey>.
- [2] R. Walden, "Analog-to-digital converter survey and analysis," *IEEE J. Select. Areas Commun.*, vol. 17, no. 4, pp. 539–550, Apr. 1999.
- [3] G. Fettweis, M. Dörpinghaus, S. Bender, L. Landau, P. Neuhaus, and M. Schlüter, "Zero crossing modulation for communication with temporally oversampled 1-bit quantization," in *Proc. 53rd Asilomar Conf. on Signals, Syst., and Comput.*, Pacific Grove, CA, USA, Nov. 2019, pp. 207–214.
- [4] M. Schlüter, M. Dörpinghaus, and G. P. Fettweis, "Joint phase and timing estimation with 1-bit quantization and oversampling," *IEEE Trans. Commun.*, vol. 70, no. 1, pp. 71–86, Jan. 2022.
- [5] S. Zeitz, F. Roth, F. Gast, M. Dörpinghaus, and G. Fettweis, "Timing synchronization and detection for systems with 1-bit quantization and runlength coding," in *Proc. IEEE Int. Workshop on Signal Process. Advances in Wireless Commun. (SPAWC)*, Lucca, Italy, Sep. 2024, pp. 421–425.
- [6] F. Mann, M. Dörpinghaus, and G. Fettweis, "Frequency offset estimation with 1-bit quantization and oversampling at the receiver," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Montreal, Canada, Jun. 2025.
- [7] M. Schlüter, F. Gast, M. Dörpinghaus, and G. P. Fettweis, "ML carrier phase estimation with 1-bit quantization and oversampling," in *Proc. IEEE Statistical Signal Process. Workshop (SSP)*, Rio de Janeiro, Brazil, Jul. 2021, pp. 376–380.
- [8] F. Gast, M. Schlüter, M. Dörpinghaus, H. Halbauer, and G. P. Fettweis, "Phase noise tracking for receivers with 1-bit quantization and oversampling," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Seoul, South Korea, May 2022, pp. 1361–1366.
- [9] P. Rodriguez-Vazquez, M. E. Leinonen, J. Grzyb, N. Tervo, A. Parssinen, and U. R. Pfeiffer, "Signal-processing challenges in leveraging 100 Gb/s wireless THz," in *Proc. 2nd 6G Wireless Summit (6G SUMMIT)*, Levi, Finland, Mar. 2020, pp. 1–5.
- [10] Z. Sha and Z. Wang, "Channel estimation and equalization for terahertz receiver with RF impairments," *IEEE J. Select. Areas Commun.*, vol. 39, no. 6, pp. 1621–1635, Jun. 2021.
- [11] M. Stein, A. Mezghani, and J. A. Nossek, "A lower bound for the Fisher information measure," *IEEE Signal Processing Letters*, vol. 21, no. 7, pp. 796–799, Jul. 2014.
- [12] M. Schlüter, M. Dörpinghaus, and G. P. Fettweis, "Bounds on channel parameter estimation with 1-bit quantization and oversampling," in *Proc. IEEE 19th Int. Workshop on Signal Process. Advances in Wireless Commun. (SPAWC)*, Kalamata, Greece, Jun. 2018, pp. 1–5.
- [13] —, "Bounds on phase, frequency, and timing synchronization in fully digital receivers with 1-bit quantization and oversampling," *IEEE Trans. Commun.*, vol. 68, no. 10, pp. 6499–6513, Oct. 2020.
- [14] M. Stein, A. Kurzl, A. Mezghani, and J. A. Nossek, "Asymptotic parameter tracking performance with measurement data of 1-bit resolution," *IEEE Trans. Signal Process.*, vol. 63, no. 22, pp. 6086–6095, Nov. 2015.
- [15] G. Zeitler, G. Kramer, and A. C. Singer, "Bayesian parameter estimation using single-bit dithered quantization," *IEEE Trans. Signal Process.*, vol. 60, no. 6, pp. 2713–2726, Jun. 2012.
- [16] O. Dabeer and A. Karnik, "Signal parameter estimation using 1-bit dithered quantization," *IEEE Trans. Inform. Theory*, vol. 52, no. 12, pp. 5389–5405, Dec. 2006.
- [17] O. Dabeer and U. Madhow, "Channel estimation with low-precision analog-to-digital conversion," in *Proc. IEEE Int. Conf. on Commun.*, Cape Town, South Africa, May 2010, pp. 1–6.
- [18] M. Windisch and G. Fettweis, "Performance analysis for blind I/Q imbalance compensation in low-IF receivers," in *Proc. First Int. Symp. on Control, Commun. and Signal Process.*, Hammamet, Tunisia, 2004, pp. 323–326.
- [19] M. Windisch, *Estimation and Compensation of I/Q Imbalance in Broadband Communications Receivers*, ser. Beiträge aus der Informationstechnik. Dresden: Vogt, 2007.
- [20] W. Gappmair and O. Koudelka, "CRLB and low-complex algorithm for joint estimation of carrier frequency/phase and IQ imbalance in direct-conversion receivers," in *Proc. 8th Int. Symp. on Commun. Syst., Netw. & Digit. Signal Process. (CSNDSP)*, Poznan, Poland, Jul. 2012, pp. 1–5.
- [21] G. Xing, M. Shen, and H. Liu, "Frequency offset and I/Q imbalance compensation for direct-conversion receivers," *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 673–680, Mar. 2005.
- [22] Y.-C. Pan and S.-M. Phoong, "A time-domain joint estimation algorithm for CFO and I/Q imbalance in wideband direct-conversion receivers," *IEEE Trans. Wireless Commun.*, vol. 11, no. 7, pp. 2353–2361, Jul. 2012.
- [23] D. Maltera and F. Sterle, "ML estimation of receiver IQ imbalance parameters," in *Proc. Int. Waveform Diversity and Design Conf.*, Pisa, Italy, Jun. 2007, pp. 160–164.
- [24] C. Zhang, Z. Xiao, B. Gao, L. Su, and D. Jin, "Receiver IQ imbalance estimation via pure noise for 60 GHz systems," *Wireless Pers. Commun.*, vol. 75, no. 2, pp. 915–924, Mar. 2014.
- [25] A. Tarighat, R. Bagheri, and A. Sayed, "Compensation schemes and performance analysis of IQ imbalances in OFDM receivers," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 3257–3268, Aug. 2005.
- [26] M. Windisch and G. Fettweis, "Blind I/Q imbalance parameter estimation and compensation in low-IF receivers," in *Proc. First Int. Symp. on Control, Commun. and Signal Process.*, Hammamet, Tunisia, Mar. 2004, pp. 75–78.
- [27] G. Fettweis, M. Löhning, D. Petrovic, M. Windisch, P. Zillmann, and W. Rave, "Dirty RF: A new paradigm invited paper," in *Proc. IEEE 16th Int. Symp. on Pers., Indoor and Mobile Radio Commun. (PIMRC)*, vol. 4, Berlin, Germany, Jul. 2005, pp. 2347–2355.
- [28] L. Giugno, V. Lottici, and M. Luise, "Efficient compensation of I/Q phase imbalance for digital receivers," in *Proc. IEEE Int. Conf. on Commun. (ICC)*, vol. 4, Seoul, South Korea, Aug. 2005, pp. 2462–2466.
- [29] S. M. Kay, *Fundamentals of Statistical Signal Processing. 1: Estimation Theory*, 20th ed., ser. Fundamentals of Statistical Signal Processing. Upper Saddle River, NJ: Prentice Hall PTR, 2013, no. 1.