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PERFORMANCE COMPARISON OF OFDM TRANSMISSION AFFECTED BY PHASE NOISE WITH AND WITHOUT PLL

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Abstract. Oscillator phase noise present in OFDM systems limits the performance by producing a common phase error and additional intercarrier interference. While phase noise is most often modelled as a Wiener process corresponding to free running oscillators, we use a recently developed approach [1] to describe the stochastical behaviour of phase locked loops (PLL) to compare the performance degradation of OFDM-transmission for a receiver operating with a free running oscillator to one using a first order charge-pump PLL having a VCO with the same linewidth. In addition to the relative linewidth of the oscillator with respect to subcarrier spacing, a characteristic quantity determining the performance is the time constant of the PLL with respect to the OFDM symbol duration. Comparing the performance for different QAM constellations it is found, that a PLL will improve bit and symbol error rate, if its gain can be made high enough to obtain time constants smaller than the OFDM symbol duration.

1. INTRODUCTION

The application of OFDM has become widespread in wireless communication systems, because of its flexibility to adapt transmission rates, its high spectral efficiency and its robust behaviour with respect to multipath effects. The price to be paid is increased sensitivity with respect to nonlinearities in the transmission channel arising from a high peak to average power ratio, non-ideal synchronization and phase noise.
Phase noise in OFDM has been studied by several authors (see e.g. [2]) predominantly considering free-running oscillators, thus modelling phase noise as a Wiener process. Analysis of the demodulated signal shows that generally phase noise leads to a common symbol rotation on all subcarriers (common phase error, CPE) which can be corrected using known pilot symbols. In addition residual intercarrier interference (ICI) occurs which is approximately Gaussian and remains uncorrected. It limits the performance, because an effective SNR ultimately determined by phase noise (when all other noise contributions become small) is left which explains the occurrence of a bit error floor.
A question one might ask in this context is, to which degree a PLL used for tracking the carrier phase will change the performance observed in OFDM transmission and under which condition the above-mentioned Wiener process is or is *not* an appropriate model for phase noise in OFDM. We address this problem within the framework of the phase noise theory which has recently been developed by *Demir* [3] et al. and *Mehrotra* [1]. In that work a thorough characterization of noise processes in

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free-running oscillators is provided as well as an extension to the behaviour of PLLs which we use to generate a well-defined stochastical phase noise process for a PLL in an OFDM transmission chain.

In the following we consider the combined effect of phase noise introduced in the analog frontend and the post-FFT correction of the common phase error as depicted in Fig. 1. To concentrate on these two effects, we make the assumption, that the time-discrete OFDM signal is directly downconverted, received and sampled with perfect symbol timing, i.e. further sampling clock or frequency synchronization errors are excluded. To compare bit and symbol error rate performance with a free oscillator to that of an OFDM receiver with a PLL, a free-running oscillator replaces the PLL in the model sketched in Fig. 1.

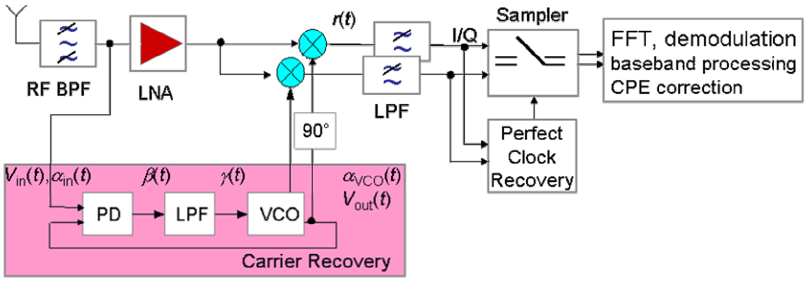

Fig. 1: OFDM receiver model with a PLL for carrier phase tracking.

The outline of the paper is as follows: In section 2 we introduce the baseband model for the OFDM system under investigation and specify how phase noise affects the received signal. Section 3 describes the stochastic models used for a free oscillator and a charge-pump PLL (which serves as an example PLL) together with asymptotic expressions for their spectra. Symbol and bit error rate curves are presented in section 4 for uncoded and coded transmission using different QAM-constellations. Given the same oscillator linewidths of free-running oscillator and VCO we illustrate, that a PLL only makes a difference, if its tracking speed is high enough to suppress phase noise on the time scale of an OFDM symbol.

2. OFDM TRANSMISSION MODEL

We study the properties of OFDM transmission. First the input bit stream is mapped to QAM-symbols. Performing an inverse Fourier transform on a group of N such QAM-symbols the time domain signal which occupies the system bandwidth $W = 1/T_u$ is obtained. The complex envelope of the transmitted signal within one OFDM symbol interval $t \in [0, T_u]$ can be written as $s(t) = \sum_{k=0}^{N-1} S(k) e^{j2\pi f_k t}$, where $f_k = k\Delta f_{car} = kW/N$ denotes the k^{th} carrier frequency and $S(k)$ the data symbol modulating the k^{th} carrier. The corresponding discrete time sequence reads

$$s(n) = \sum_{k=0}^{N-1} S(k) e^{j2\pi kn/N} \quad , \qquad (1)$$

to which a cyclic prefix of length T_{GI} is added. The received sampled OFDM signal $r(n) = s(n)e^{j\phi(n)} + \xi(n)$ is disturbed by AWGN samples $\xi(n)$ (a non-dispersive channel is assumed to concentrate on the influences of PLL and CPE correction) and the phase noise process $\phi(n)$ that causes a random phase modulation.

Due to phase noise up- and down conversion oscillator signals cannot be described by a perfectly harmonic carrier signal $x_c(t) = e^{j2\pi f_c t}$ (with carrier frequency f_c). Instead a random phase shift $\phi(t)$ which will be described in terms of a random time shift $\alpha(t)$ (with respect to an ideal oscillator) has to be taken into account producing the imperfect oscillator signal $x_c(t+\alpha(t)) = e^{j2\pi f_c (t+\alpha(t))}$. The demodulated complex carrier amplitude received on the l^{th} subcarrier in an OFDM symbol becomes:

$$R_l = \frac{1}{N}\sum_{n=0}^{N-1} r(n)e^{-j2\pi nl/N} = \frac{1}{N}\sum_{n=0}^{N-1}\left[\sum_{k=0}^{N-1} S(k)e^{j2\pi kn/N}\right]e^{j\phi(n)}e^{-j2\pi nl/N} + N_l \ . \qquad (2)$$

N_l still represents the additive gaussian noise, the properties of which are unaffected by the demodulation. The sum in eq. (2) can be split into interference created by the common phase error term and a remaining sum of intercarrier interference:

$$R_l = \underbrace{S(l)\frac{1}{N}\sum_{n=0}^{N-1} e^{j\phi(n)}}_{CPE} + \underbrace{\frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=0,k\neq l}^{N-1} S(k)e^{-j2\pi(k-l)n/N}e^{j\phi(n)}}_{ICI} + N_l \ . \qquad (3)$$

We assume ideal CPE correction which is equivalent to filtering the phase noise spectrum $S_{osc}(f)$ by a low pass filter with transfer function $H(f) = \mathrm{sinc}^2(f/\Delta f_{car})$. In that case the remaining ratio between intercarrier interference and signal power is [4]

$$P_{ICI}/S = \sum_{k\neq l}\int_0^\infty \mathrm{sinc}^2(f/\Delta f_{car} - (k-l))\, S_{osc}(f)\, \mathrm{d}f \ . \qquad (4)$$

Using this result an effective SNR (with respect to an AWGN channel) can be calculated approximately as:

$$\left(\frac{S}{P_{ICI}+P_{AWGN}}\right)_l \cong \frac{1}{\int_{-\infty}^{\infty}\left[1\text{-}\mathrm{sinc}^2(f/\Delta f_{car})\right]S_{osc}(f)df + 1/SNR_{AWGN}} \ . \qquad (5)$$

In section 3 we will apply this argument to study the conditions under which a PLL improves the performance with respect to a free-running oscillator.

3. PHASE NOISE MODEL

To quantify the influence of phase noise we use the baseband representations of stochastic phase processes for a free-running oscillator and a charge-pump PLL (CP-PLL) described in the two following subsections.

Power Spectral Density and Phase Process of a free-running Oscillator

Demir et al. presented in [3] a phase noise theory for free-running oscillators which are asymptotically described by a Wiener process. The time shift $\alpha(t)$ of the oscillating signal with respect to an ideal reference is described as $\alpha(t) = \sqrt{c}B(t)$, where $B(t)$ represents a standard Wiener process [5]. The variance of the process $\alpha(t)$ thus grows linearly in time proportional to a constant c which describes the oscillator quality. The Lorentzian spectrum associated with a Wiener process can be specified by a single parameter, its 3 dB bandwidth (which is related to c by $\Delta f_{3dB} = \pi c f_c^2$):

$$S^{free}(f) = \frac{1}{\pi} \cdot \frac{\Delta f_{3dB}}{\Delta f_{3dB}^2 + f^2} \quad . \tag{6}$$

The phase process of the free-running oscillator $\phi(t) = 2\pi f_c \alpha(t)$ is described as the integral of a zero-mean unit variance gaussian R.V. $\xi \sim \mathcal{N}(0,1)$:

$$\phi(t) = 2\pi f_c \sqrt{c_{free}} \int_0^{B(t)} dB(\tau) = 2\pi f_c \sqrt{c_{free}} \int_0^t \xi(\tau) d\tau \ . \tag{7}$$

Power Spectral Density and Phase Process of a Charge-Pump PLL

The basic structure of a PLL is shown in Fig. 1, where the reference signal V_{in} with time shift $\alpha_{in}(t)$ is to be tracked. The reference signal exhibits the noise spectrum of the oscillator by which it was generated which we assume has good quality (at least as good as the receiving LO).

The spectrum of the PLL output signal V_{out} (with time shift $\alpha_{VCO}(t)$) will be determined at high offset frequencies (relative to the fundamental oscillation frequency) by the properties of the VCO. The characteristics of the VCO can be obtained as the open loop spectrum of the PLL. If the PLL is locked, it follows at low frequencies the spectrum of the reference signal with a transition frequency defined by the loop bandwidth. For our investigation the spectrum of interest, is the spectrum of the signal at the phase detector output, denoted $\beta(t)$, which affects the signal after down-conversion.

To obtain a description of the stochastical process at the output of a PLL, we follow the approach outlined in [1] which starts from the definition of $\beta(t)$:

$$\beta(t) = \alpha_{VCO}(t) - \alpha_{in}(t) \, . \tag{8}$$

PLL analysis proceeds by solving the associated stochastical differential equation (SDE) for $\beta(t)$ coupled to the time shift at the low pass filter output $\gamma(t)$. The spectrum is derived from the autocorrelation function of this output signal which is asymptotically independent of t, because the PLL output corresponds to a wide-sense stationary stochastic process.

For further details the reader is referred to [1]. Here we only want to outline, how we generated stochastical processes for typical PLLs. As an example we use the generation of such a process for the particular case of a charge-pump PLL. For a CP-PLL the system of coupled SDEs for $\beta(t)$ and $\gamma(t)$ can be found in [1]:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = -\underbrace{\begin{bmatrix} 0 & -\sqrt{c_{PLL}} \\ \omega_1 & \sqrt{c_{PLL}} \end{bmatrix}}_{A} \cdot \begin{bmatrix} \beta \\ \gamma \end{bmatrix} + \underbrace{\begin{bmatrix} \sqrt{c_{VCO}} & -\sqrt{c_{in}} \\ -\sqrt{c_{VCO}} & \sqrt{c_{in}} \end{bmatrix}}_{D} \begin{bmatrix} \xi_{VCO} \\ \xi_{in} \end{bmatrix} \ . \tag{9}$$

The quantities in eq. (9) have the following meaning: $\sqrt{c_{PLL}}$ with dimension [s^{-1}] is defined as $\sqrt{c_{PLL}} = k_{PD}\sqrt{c_{control}}$, where k_{pd} is the phase detector gain, $c_{control}$ characterizes noise sources in the control node of the VCO (phase detector plus low pass filter) and $\bar{\gamma} = \gamma / k_{PD}$. The parameters c_{VCO} and c_{in} characterize the strength of noise sources of VCO and input signal, respectively. Finally, ξ_{VCO} and ξ_{in} are uncorrelated white noise sources and ω_1 specifies the corner frequency of the loop filter.
The asymptotic spectrum is governed by the eigenvalues of the matrix A which are determined to be $\lambda_{1,2} = \left(\sqrt{c_{PLL}} \pm \sqrt{c_{PLL} - 4\omega_1\sqrt{c_{PLL}}}\right)/2 = \left(\sqrt{c_{PLL}} \pm \sqrt{\varepsilon}\right)/2$.

Similarly to ordinary first order differential equations these eigenvalues play the role of time constants which determine the speed of the PLL response. In addition certain coefficients $\mu_{1,2}$, $\nu_{1,2}$ are used in [1][1] to express the asymptotic spectrum (eq.(13) in [1]). If one is only interested in checking the correctness of the numerically generated spectrum of the stochastical process, the first order approximation (which is already very precise) can be expressed also by the circuit parameters themselves. The power spectral density of the VCO output of the charge-pump PLL reads

$$\begin{aligned} S_{\alpha\alpha}(\Delta\omega) = \omega_c^2 \exp\left\{-\omega_c^2 (c_{VCO} + c_{in}) \frac{\sqrt{c_{PLL}} - 2\omega_1}{2\varepsilon}\right\}\Bigg[\frac{c_{in}}{\Delta\omega^2} + \ldots \\ + \frac{c_{VCO}(\lambda_1 - \omega_1) - c_{in}(\lambda_1 + \omega_1)}{\sqrt{\varepsilon}(\lambda_1^2 + \Delta\omega_1^2)} + \frac{c_{VCO}(-\lambda_2 + \omega_1) - c_{in}(\lambda_2 + \omega_1)}{\sqrt{\varepsilon}(\lambda_2^2 + \Delta\omega_1^2)}\Bigg] \end{aligned} . \qquad (10)$$

The relevant spectrum for the reception of OFDM signals at the output of the mixer is obtained from the AKF of $\beta(t)$ and equals the sum of two Lorentzians:

$$S_{\beta\beta}(\Delta\omega) = \omega_c^2 \frac{c_{VCO} + c_{in}}{\sqrt{c_{PLL} - 4\omega_1\sqrt{c_{PLL}}}} \cdot \left[\frac{\lambda_1 - \omega_1}{\lambda_1^2 + \Delta\omega^2} - \frac{\lambda_2 - \omega_1}{\lambda_2^2 + \Delta\omega^2}\right] . \qquad (11)$$

Influence of CPE Correction on the Spectra of free-running Oscillator and CP-PLL

Typical spectra for a CP-PLL are shown on the left hand side in Fig. 2, where the asymptotic behaviour at the VCO output of the PLL and mixer outputs (equivalent to phase detector output) according to eqs. (10, 11) are shown as thick lines and compared to numerically calculated '*noisy*' looking spectra obtained from integrating eq. (9) with the Euler-Maruyama method [5] averaging over several realisations of the power spectrum of the stochastical process. The PLL output spectrum $S_{\alpha\alpha}(\Delta\omega)$ follows at low frequencies the spectrum of the high-quality input signal and coincides after a bump with the open-loop VCO output spectrum at high frequencies which falls off with -20 dB/decade. The spectrum $S_{\beta\beta}(\Delta\omega)$ of the difference signal

[1] A factor of 1/2 which appears erroneously in the second term for ν_1 in the appendix of [1] has been omitted.

$\beta(t)$ is a high-pass filtered version of the VCO output spectrum $S_{\alpha\alpha}(\Delta\omega)$ filtering out the phase noise at low frequencies.

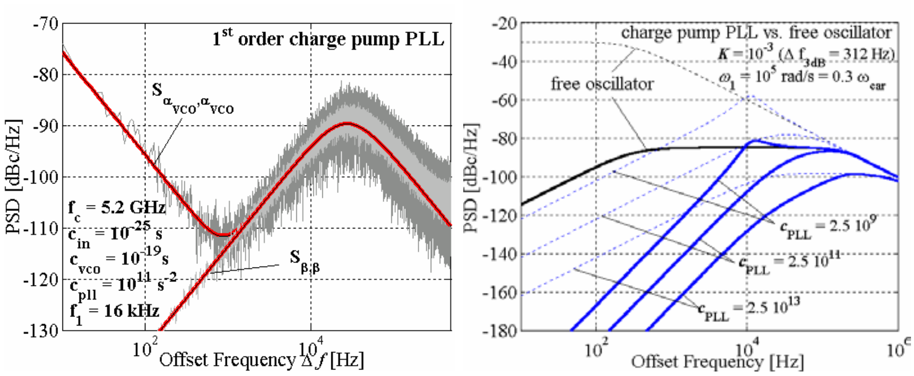

Fig. 2: Comparison of numerically generated power spectral densities at the PLL and phase detector outputs with the analytical expressions (left). Comparison between phase noise spectra of a free oscillator and a CP-PLL before (dashed lines) and after (full lines) multiplication with the ICI weighting function (CPE correction).

To understand under which circumstances use of a PLL will alter OFDM performance, one has to note, that a similar effect is obtained with CPE correction, as shown on the right hand side of Fig. 2. We compare weighted (thick lines) and unweighted (dashed lines) spectra of a free oscillator and a PLL. The 3dB frequencies of VCO and free oscillator are set in both cases to $\Delta f_{3\text{dB}}$ = 312 Hz ($K = 10^{-3}$). Weighting the phase noise spectrum with $1-\text{sinc}^2(f/\Delta f_{car})$ removes the low-frequency part of the spectrum which masks the differences between free oscillators with CPE correction and PLLs, if the gain in the phase locked loop is not sufficiently high (its time constant sufficiently small). Setting the parameter Δf_{car}, to 312.5 kHz, appropriate for WLANs, implicitly introduces a time scale, which corresponds to T_{u} = 3.2 µs. Therefore three different gain values of the phase detector are shown which is equivalent to changing the time constant of the phase noise processes generated at the phase detector output of the PLL. For the particular values here, the time constants are given approximately by $1/\lambda$= 40, 2.76 and 0.20 µs (defining a relative time constant $\tau = 1/(\lambda \cdot T_u)$ this corresponds to τ = 12.5, 0.86 and 0.064). Only the two smaller ones improve the performance with respect to the free-running oscillator.

4. PERFORMANCE EXAMPLE FOR WLAN-RELATED PARAMETERS

To study quantitatively the influence of different local oscillators with and without PLL on OFDM transmission we simulated a typical WLAN system with 64 carriers per OFDM symbol. The transmitted signal was disturbed by additive white Gaussian noise plus phase noise. Stochastical sample processes for free running oscillators and charge-pump PLLs were generated according to the method described in section 3. The effect of varying relative oscillator linewidth $K = \Delta f_{3\text{dB}}/\Delta f_{\text{car}}$ is seen in Fig. 3.

We changed the parameter c_{free} defined in eq.(7) for the free oscillator and used the same c-values for c_{VCO} in the charge-pump PLL with fixed values of $\omega_1 = 10^5$ rad/s comparing a relatively low gain of the PLL with $c_{\mathrm{PLL}} = 2.5\cdot 10^{10}$ s^{-2} ($1/\lambda = 12.65$ µs, $\tau = 3.95$) to one that is 3 orders of magnitude larger ($1/\lambda = 0.2$ µs, $\tau = 0.064$).

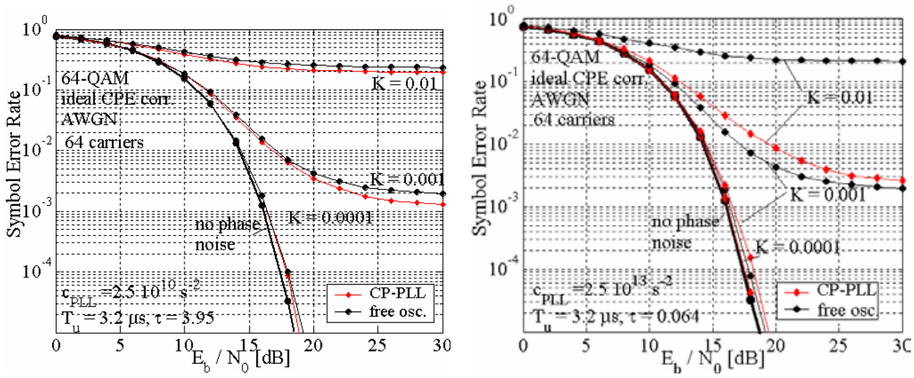

Fig. 3: Comparison of CP-PLL and free oscillator for uncoded 64-QAM transmission for relative linewidths $K = 10^{-2}\dots 10^{-4}$ for slow (left) and fast (right) tracking.

Input phase noise, determined by c_{in}, was kept fixed at 10^{-21} s. For a PLL time constant longer than the OFDM symbol duration of 3.2 µs (Fig. 3, left) the improvement with a PLL is negligible. For a relative time constant of 0.064 a PLL markedly improves performance with respect to a free running oscillator (right).

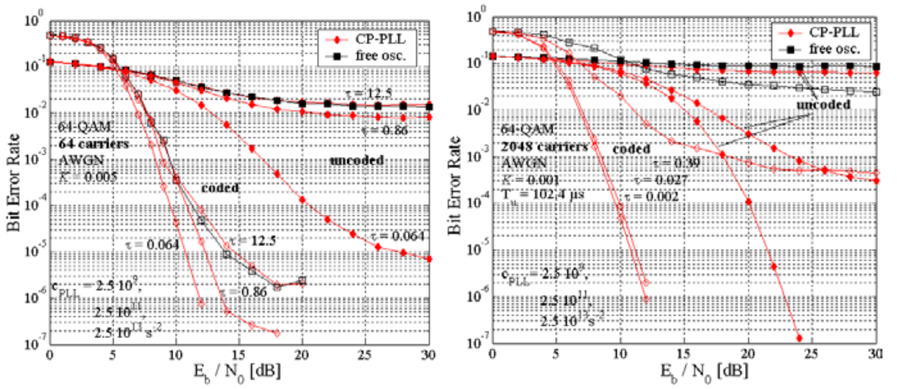

Fig. 4: Comparison of coded (open symbols) and uncoded 64-QAM transmission for several time constants of the PLL with 64 (left) and 2048 carriers (right).

The influence of the PLL time constant is further illustrated in Fig.4 for uncoded and coded 64-QAM transmission using bit-interleaved coded modulation with a convolutional code of memory 6 and rate 1/2. Coding significantly improves the performance reducing the error floor. In addition we compared the influence of OFDM symbol duration by increasing the number of carriers from 64 to 2048, applying ideal CPE correction in all cases. Although a better quality oscillator ($K = 10^{-3}$) was used for 2048 carriers, ideal CPE correction even with coding is insufficient to achieve a BER$<10^{-2}$, if the oscillator is free running. Using the same phase detector gains (c_{PLL}-values) as for 64 carriers the influence of the PLL becomes stronger, because the relative time constant is shorter, showing that for longer OFDM-symbols a carrier recovery circuit is as well more feasible and helpful. The influence of constellation size is shown in Fig. 5 for uncoded transmission. Note that the tracking capability of PLLs should benefit from the high SNR which is inherently necessary to use large constellations. Again long and short time constants ($\tau = 3.95$ / 0.064) of the PLL are compared as in Fig. 3, demonstrating, that the tracking capability of the PLL has to be brought into the time range of one OFDM symbol.

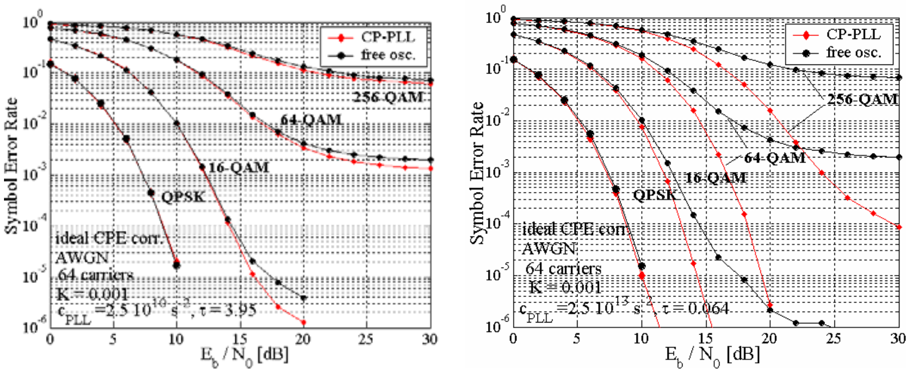

Fig. 5: Influence of constellation size for slow (left) and fast (right) tracking.

CONCLUSIONS

A stochastical phase noise model to describe the behaviour of a PLL was applied to characterize the performance of OFDM taking into account the effect of carrier tracking in an OFDM receiver. Apart from the relative linewidth of the oscillator the time constant of the phase process at the phase detector output of the PLL with respect to OFDM symbol duration becomes important. To model the performance with a free-running oscillator and CPE correction appears to be valid, as long as the time constant remains long with respect to one OFDM symbol duration. For short symbols (WLANs) and/or time variant mobile channels, where the gain in the PLL can not be made very large due to stability considerations, this is a reasonable approximation. For broadcasting standards such as DVB this might be different.

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